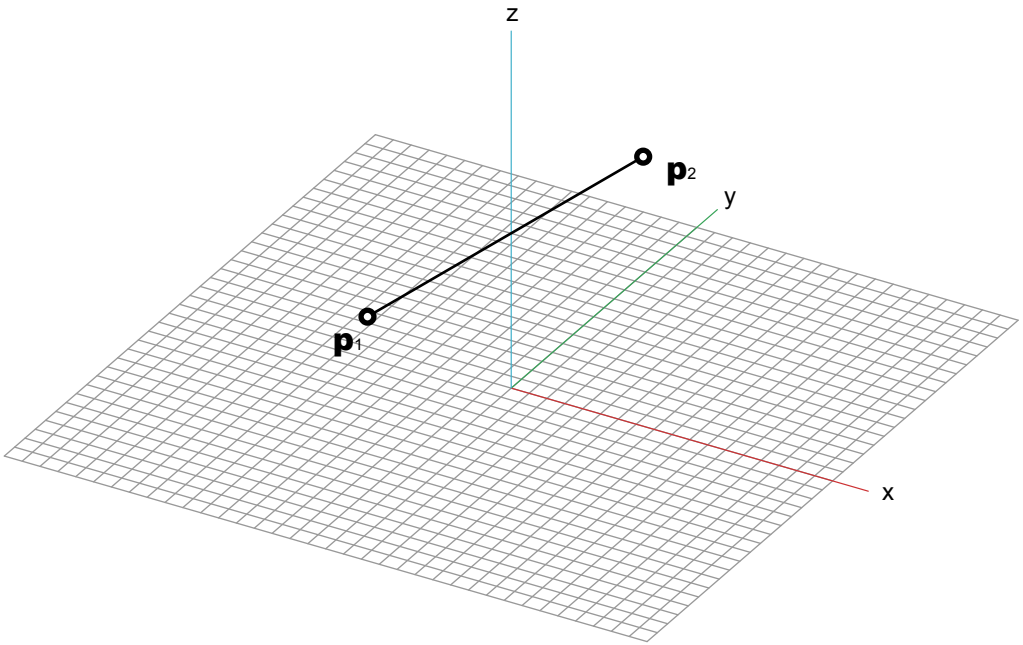
MORPH SEMINAR 2009

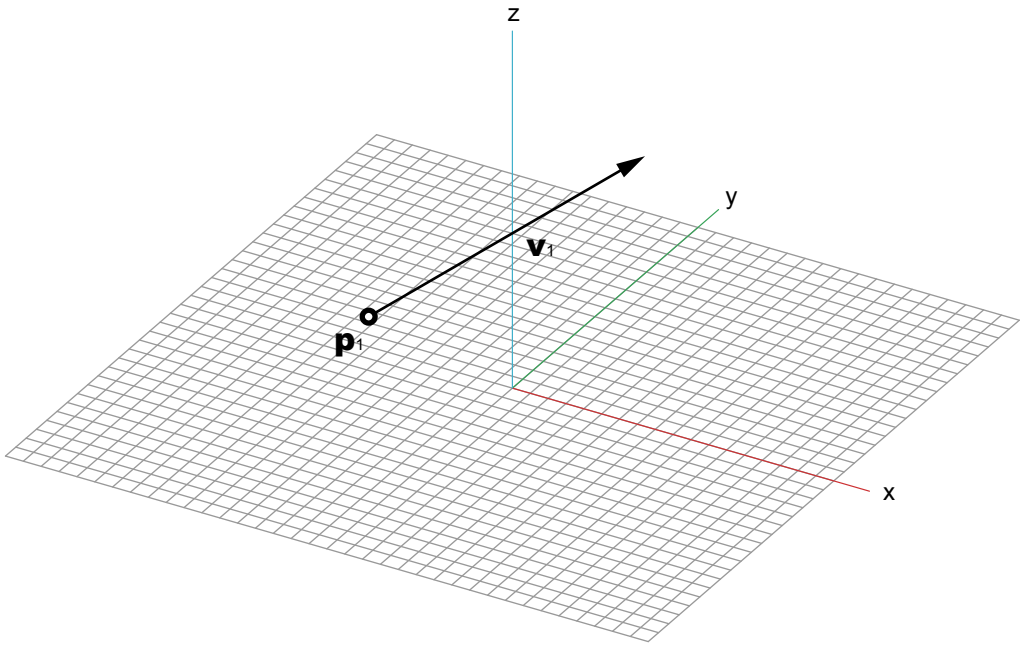
MATHEMATICS FOR SYSTEMATIC MODELING

WEEK2 LINE

DEFINITION OF LINE



Line by Two Points

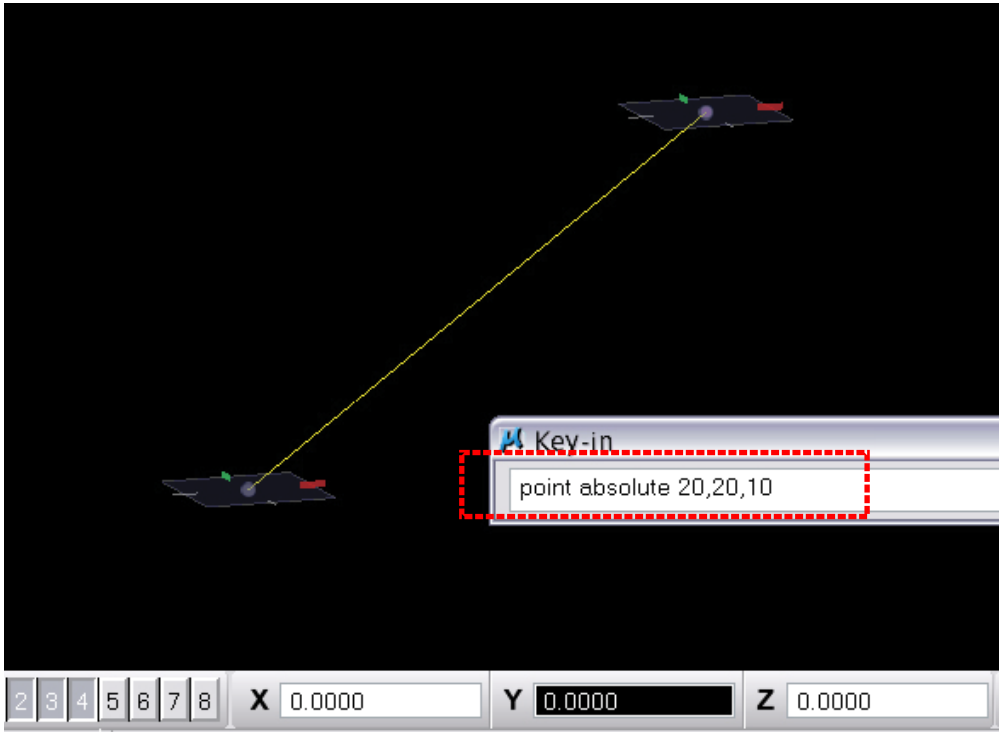


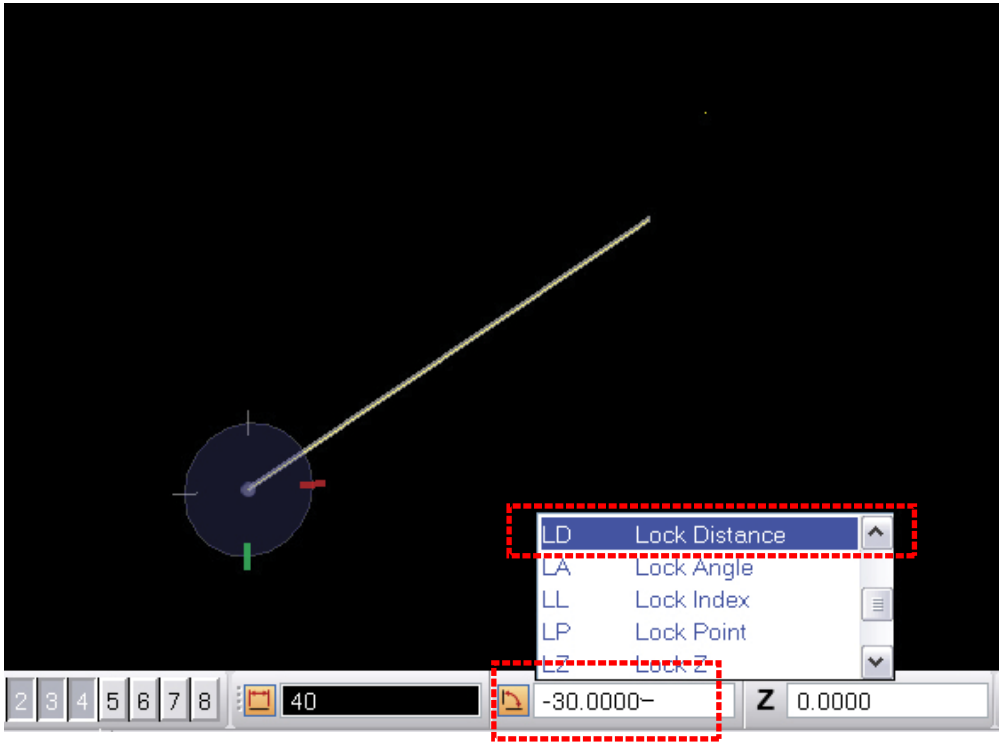
Line by One Point and One Vector

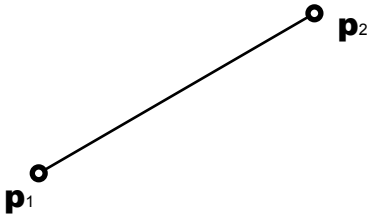
$$\mathbf{v}_1 = \mathbf{p}_2 - \mathbf{p}_1$$

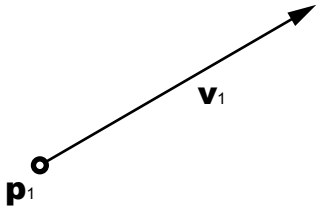
$$\mathbf{p}_2 = \mathbf{p}_1 + \mathbf{v}_1$$

LINE IN MICROSTATION

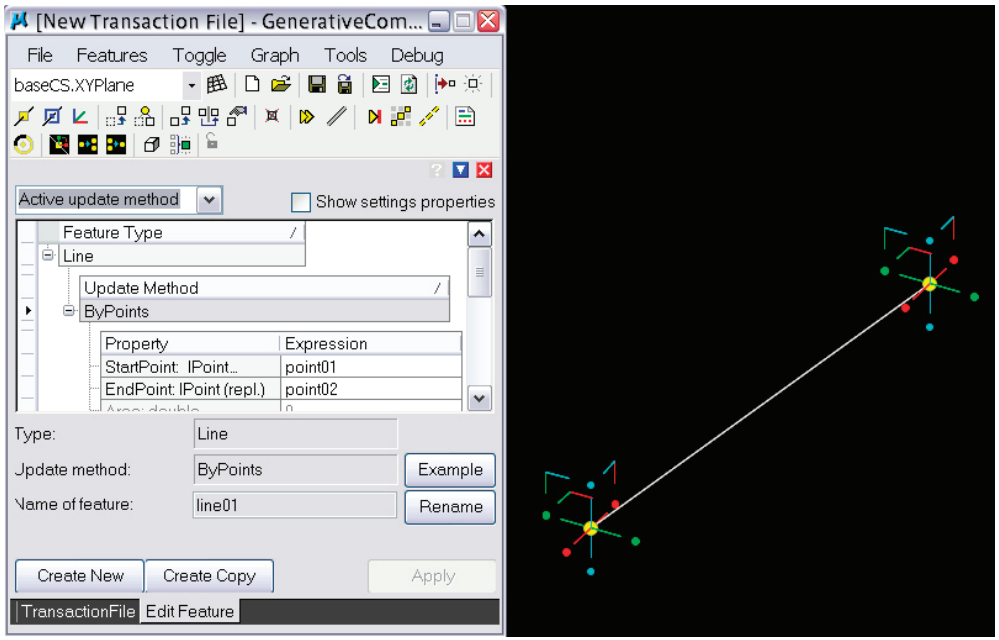


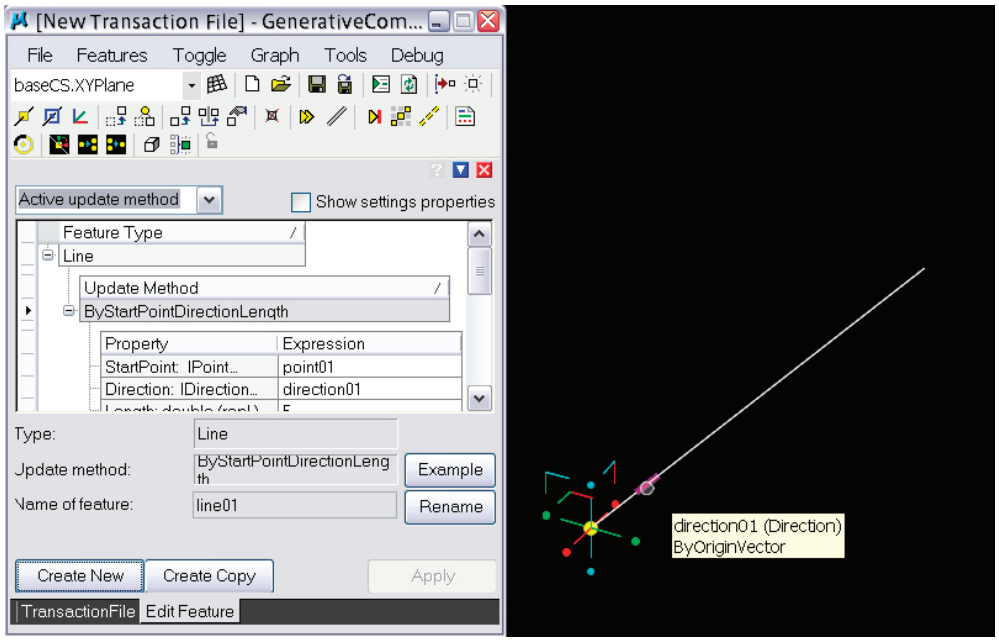


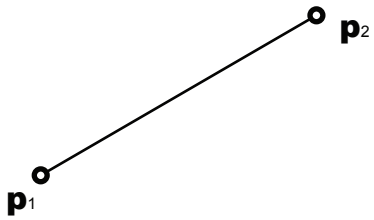


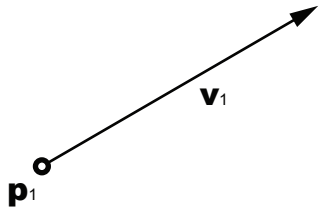


LINE IN GENERATIVE COMPONENTS

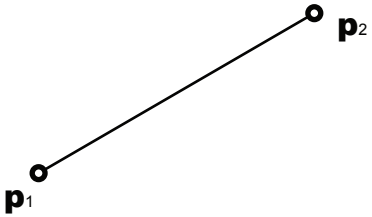


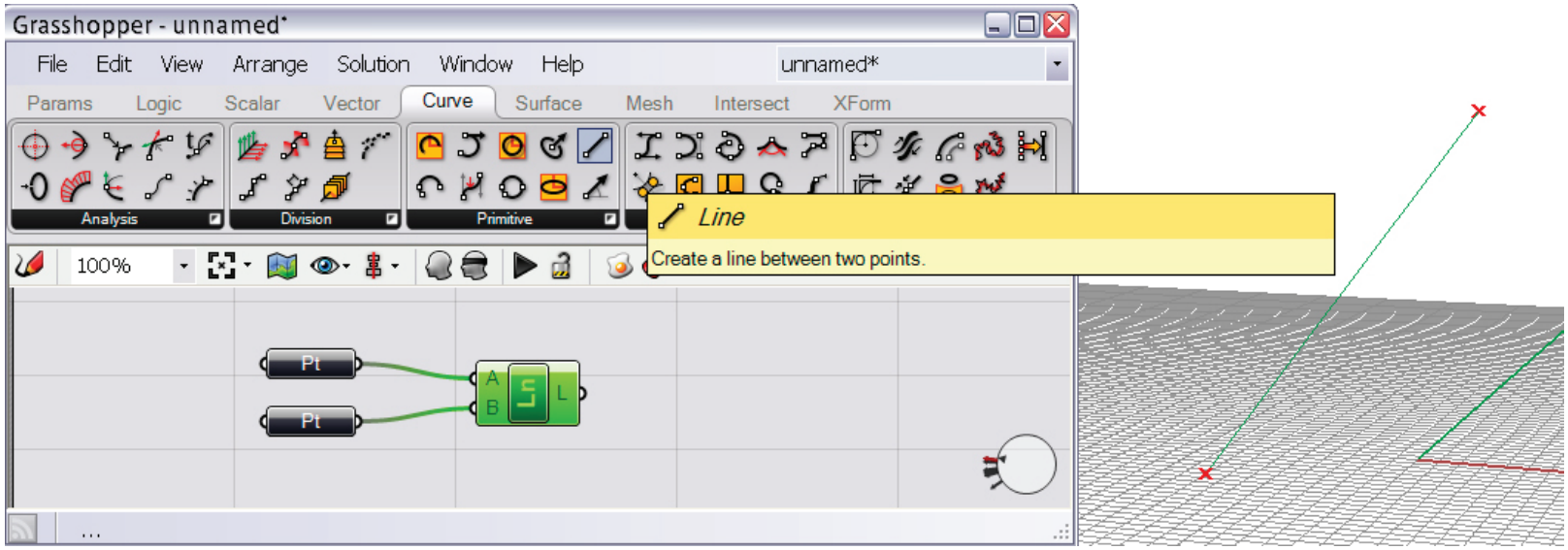


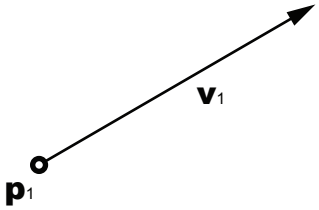


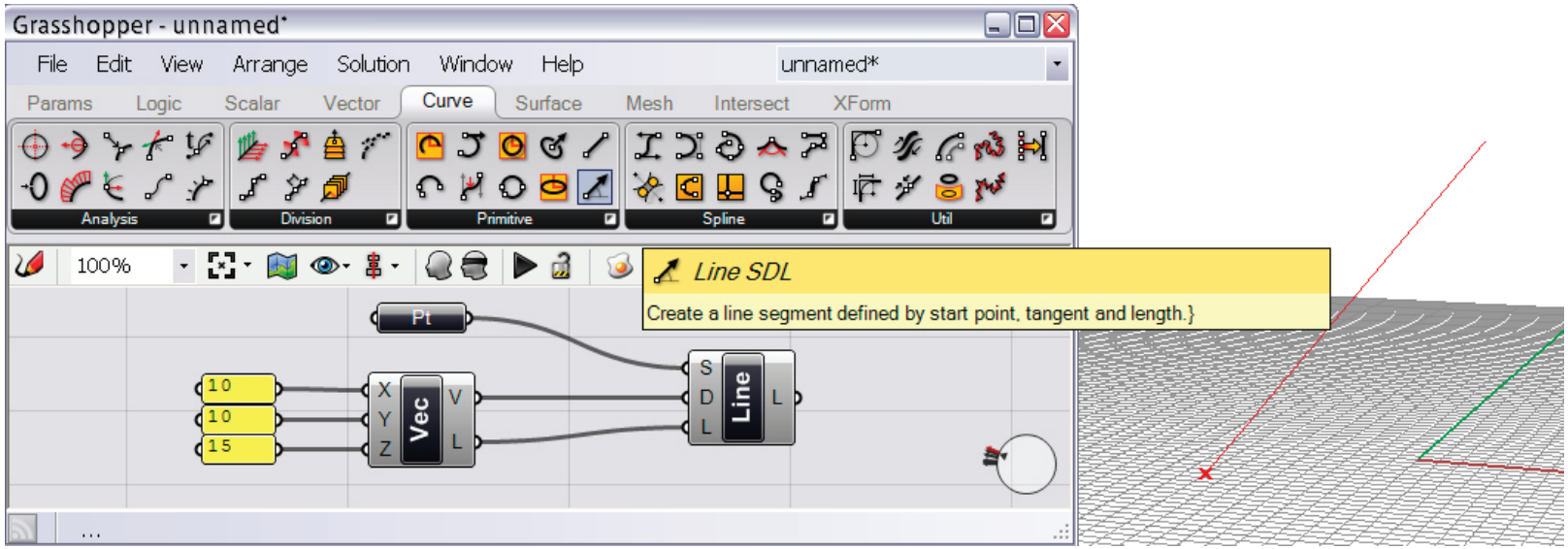


LINE IN GRASSHOPPER

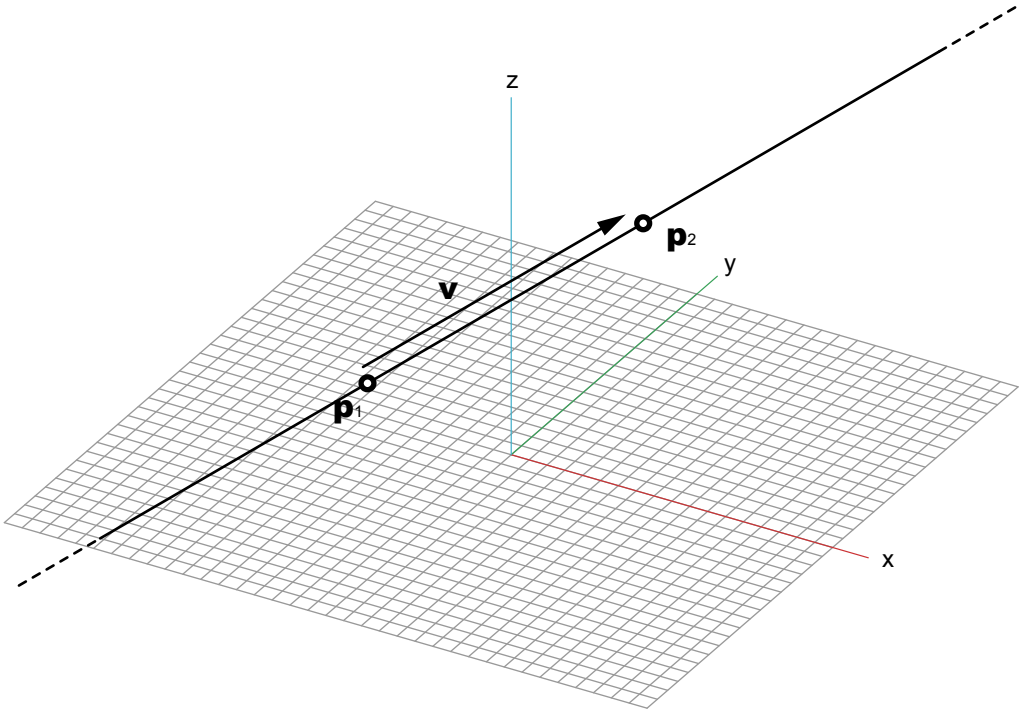




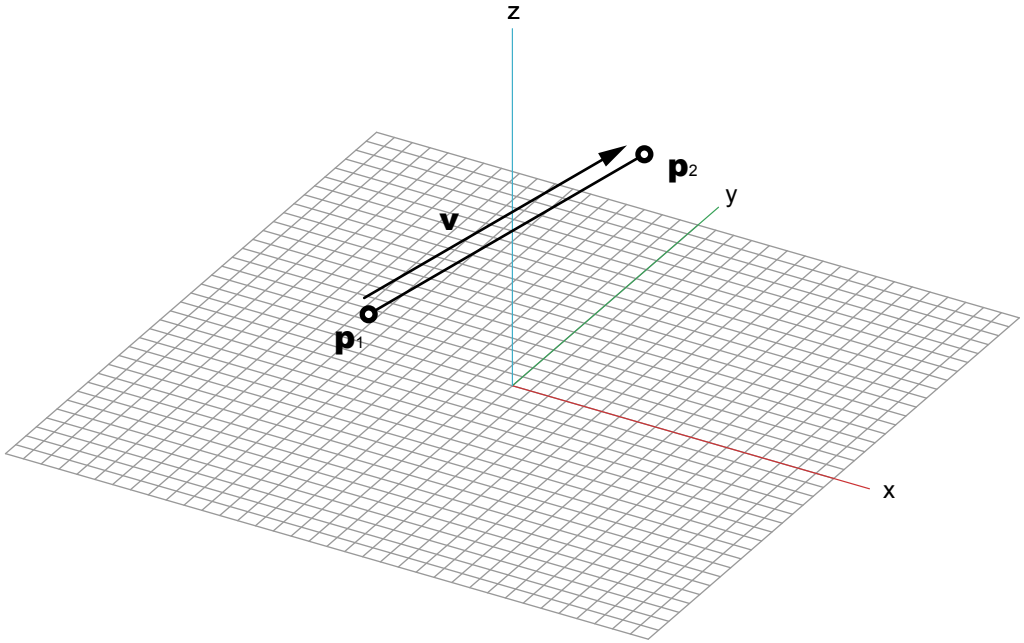




LINE SEGMENT AND INFINITE LINE



Infinite Line



Line Segment

$\mathbf{p}_1 = (p_{1x}, p_{1y}, p_{1z})$

$\mathbf{p}_2 = (p_{2x}, p_{2y}, p_{2z})$

$\mathbf{v} = (v_x, v_y, v_z)$

$= (p_{2x} - p_{1x}, p_{2y} - p_{1y}, p_{2z} - p_{1z})$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{p} = \mathbf{p}_1 + t\,\mathbf{v} = \begin{pmatrix} p_{1x} + t\,v_x \\ p_{1y} + t\,v_y \\ p_{1z} + t\,v_z \end{pmatrix}$$

$$\frac{x - p_{1x}}{p_{2x} - p_{1x}} = \frac{y - p_{1y}}{p_{2y} - p_{1y}} = \frac{z - p_{1z}}{p_{2z} - p_{1z}}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{p} = \mathbf{p}_1 + t\,\mathbf{v} = \begin{pmatrix} p_{1x} + t\,v_x \\ p_{1y} + t\,v_y \\ p_{1z} + t\,v_z \end{pmatrix} \qquad 0 < t < 1$$

$$\frac{x - p_{1x}}{p_{2x} - p_{1x}} = \frac{y - p_{1y}}{p_{2y} - p_{1y}} = \frac{z - p_{1z}}{p_{2z} - p_{1z}}$$

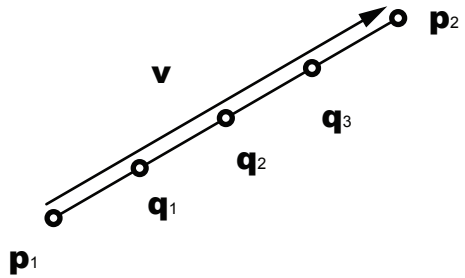
$$p_{1x} < x < p_{2x}$$

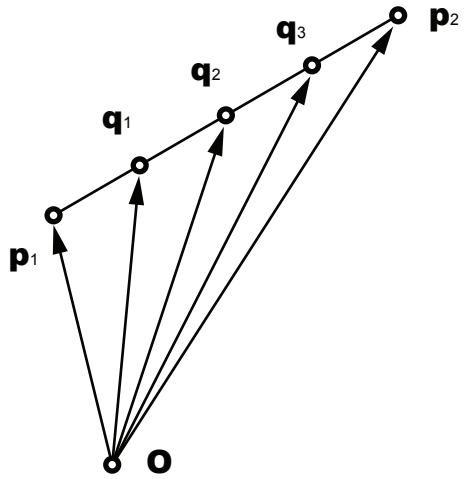
$$p_{1y} < y < p_{2y}$$

$$p_{1z} < z < p_{2z}$$

DIVISION OF LINE

Division by N





$\mathbf{q}_1 = \mathbf{p}_1 + 1/N \times \mathbf{v}$

$\mathbf{q}_2 = \mathbf{p}_1 + 2/N \times \mathbf{v}$

$\vdots$

$\mathbf{q}_i = \mathbf{p}_1 + i/N \times \mathbf{v}$

$\vdots$

$\mathbf{q}_{N-1} = \mathbf{p}_1 + (N-1)/N \times \mathbf{v}$

$\mathbf{q}_1 = (N-1)/N \times \mathbf{p}_1 + 1/N \times \mathbf{p}_2$

$\mathbf{q}_2 = (N-2)/N \times \mathbf{p}_1 + 2/N \times \mathbf{p}_2$

$\vdots$

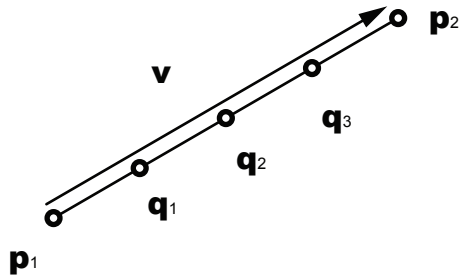
$\mathbf{q}_i = (N-i)/N \times \mathbf{p}_1 + i/N \times \mathbf{p}_2$

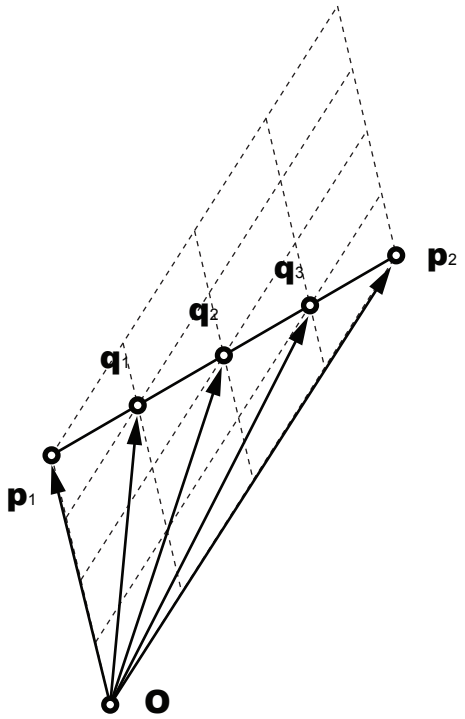
$\vdots$

$\mathbf{q}_{N-1} = 1/N \times \mathbf{p}_1 + (N-1)/N \times \mathbf{p}_2$

DIVISION OF LINE

Division by N





$\mathbf{v} = \mathbf{p}_2 - \mathbf{p}_1$

$\mathbf{q}_1 = \mathbf{p}_1 + 1/N \times \mathbf{v}$ $= (N-1)/N \times \mathbf{p}_1 + 1/N \times \mathbf{p}_2$

$\mathbf{q}_2 = \mathbf{p}_1 + 2/N \times \mathbf{v}$ $= (N-2)/N \times \mathbf{p}_1 + 2/N \times \mathbf{p}_2$

$\vdots$ $\vdots$

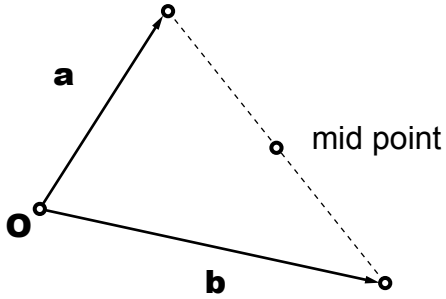
$\mathbf{q}_i = \mathbf{p}_1 + i/N \times \mathbf{v}$ $= \mathbf{p}_1 + i/N \times (\mathbf{p}_2 - \mathbf{p}_1)$ $= \mathbf{p}_1 - i/N \times \mathbf{p}_1 + 1/N \times \mathbf{p}_2$ $= (N-i)/N \times \mathbf{p}_1 + i/N \times \mathbf{p}_2$

$\vdots$ $\vdots$

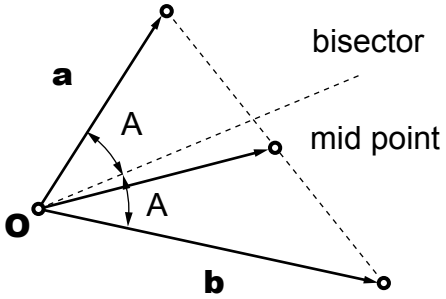
$\mathbf{q}_{N-1} = \mathbf{p}_1 + (N-1)/N \times \mathbf{v}$ $= 1/N \times \mathbf{p}_1 + (N-1)/N \times \mathbf{p}_2$

MID POINT AND BISECTOR

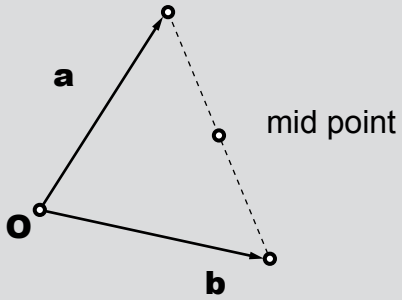
MID POINT

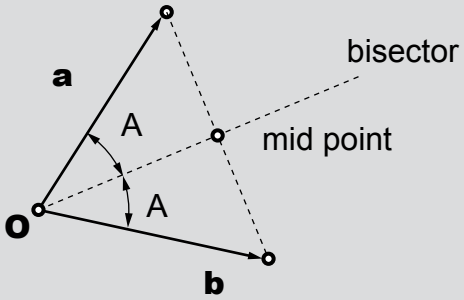


BISECTOR



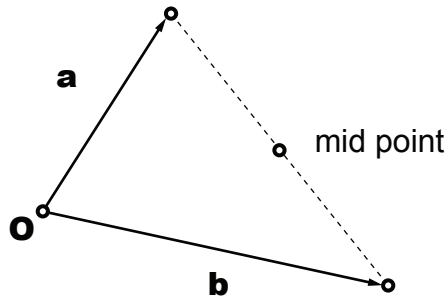
When |**a**| = |**b**|



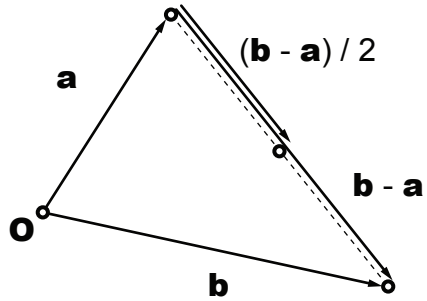


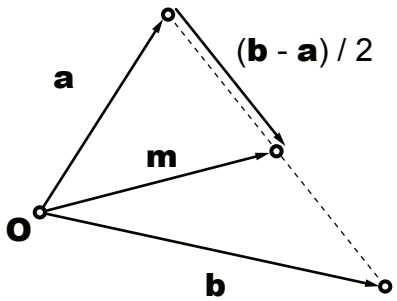
MID POINT

MID POINT



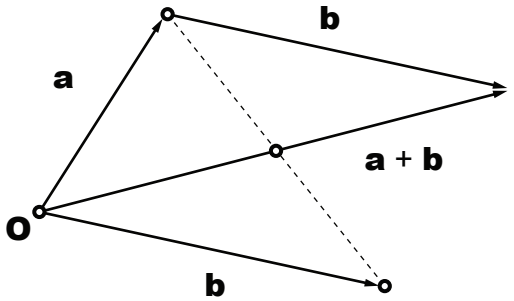
By Subtraction

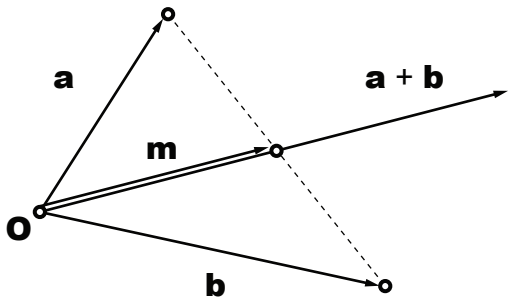




$$\mathbf{m} = \mathbf{a} + (\mathbf{b} - \mathbf{a}) / 2$$
$$= \mathbf{a} + \mathbf{b}/2 - \mathbf{a}/2$$
$$= \mathbf{a}/2 + \mathbf{b}/2$$
$$= (\mathbf{a} + \mathbf{b}) / 2$$

By Addition

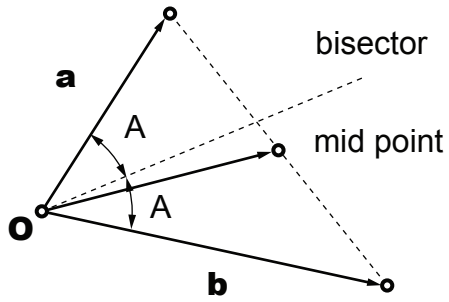




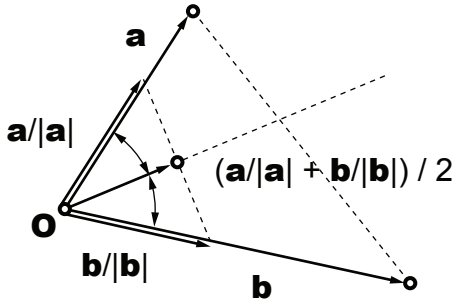
$$\mathbf{m} = (\mathbf{a} + \mathbf{b}) / 2$$

BISECTOR

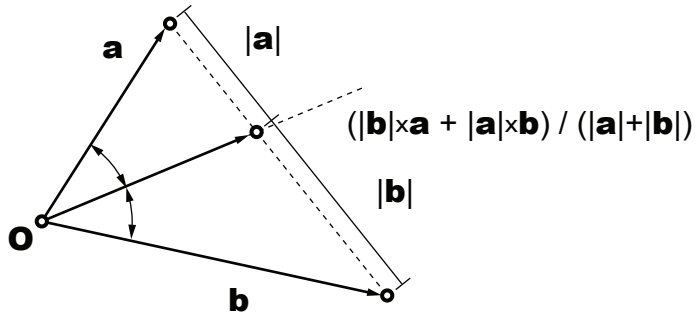
BISECTOR

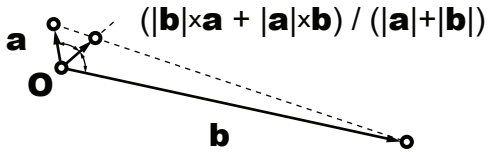


By Unitization

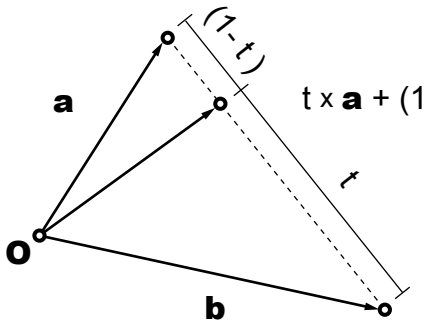


By Ratio

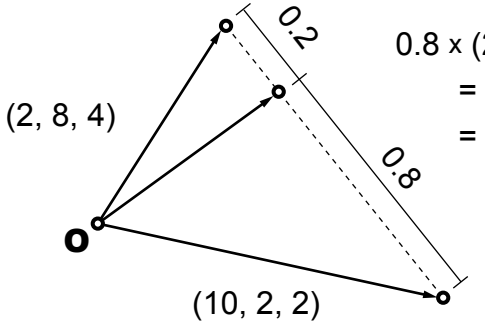




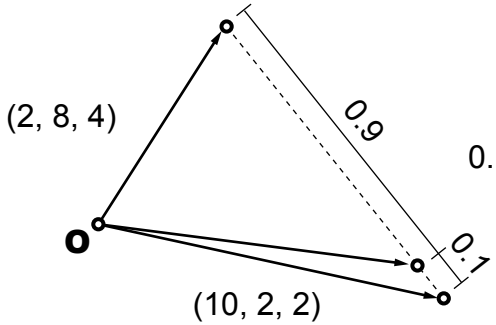
VECTOR INTERPOLATION



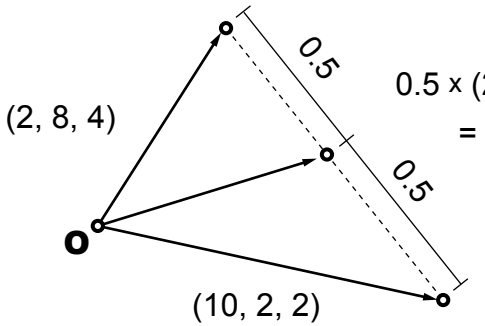
t x **a** + (1-t) x **b**



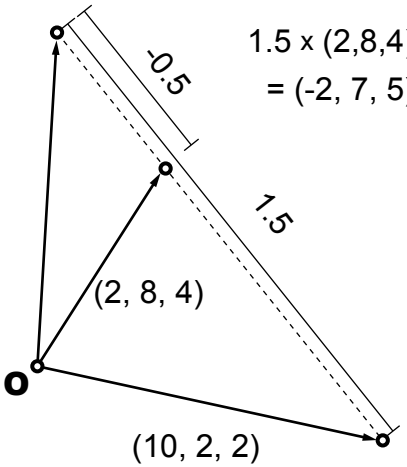
0.8 x (2,8,4) + 0.2 x (10,10,2)
= (1.6+2, 6.4+2, 3.2+0.4)
= (3.6, 8.4, 3.6)



0.1 x (2,8,4) + 0.9 x (10,10,2)
= (9.2, 9.8, 2.2)



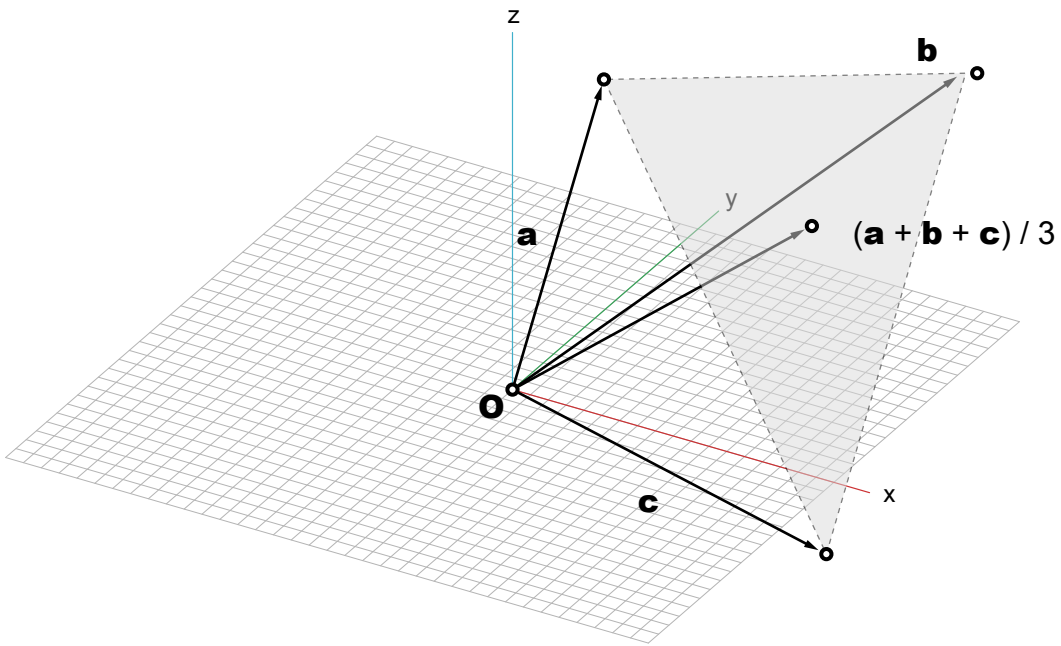
0.5 x (2,8,4) + 0.5 x (10,10,2)
= (6, 9, 3)

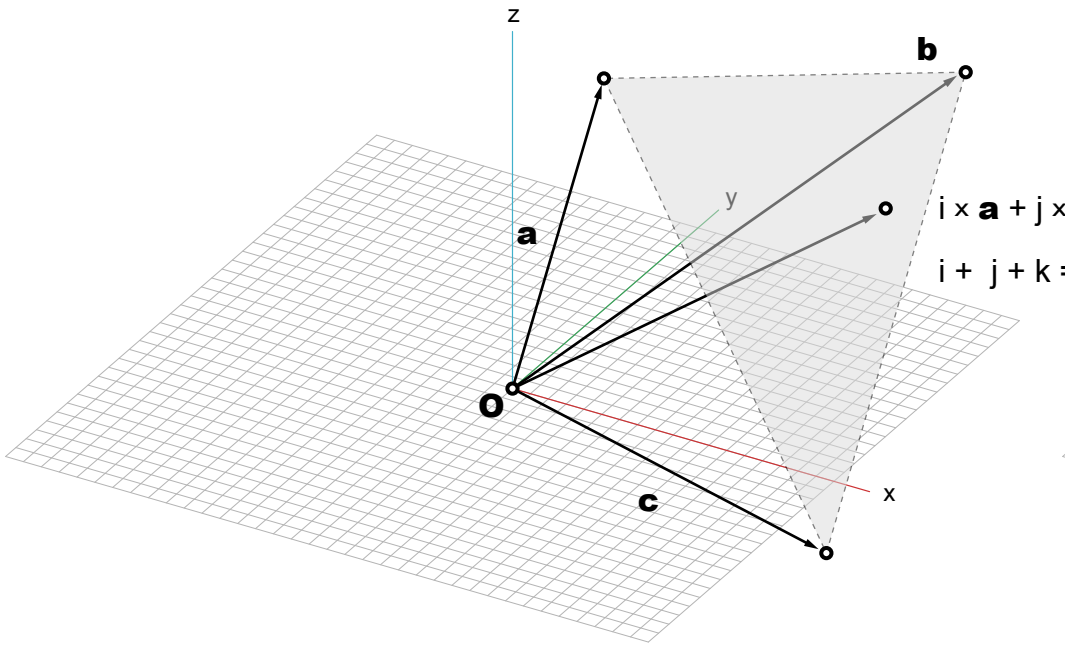


1.5 x (2,8,4) + -0.5 x (10,10,2)
= (-2, 7, 5)

(external division)

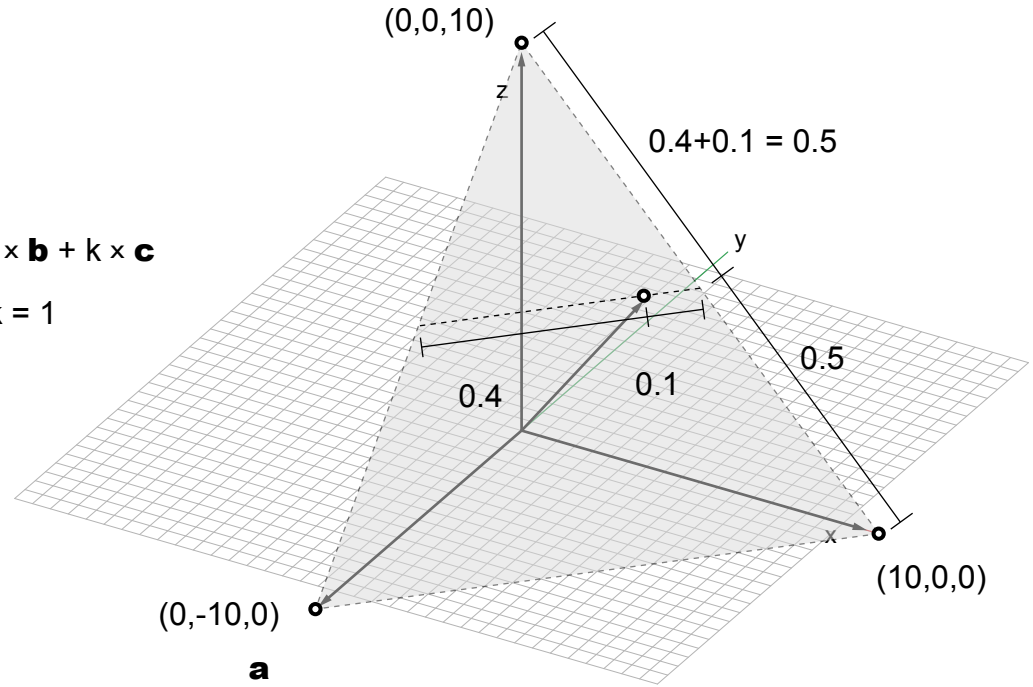
CENTER POINT AND DIVISION OF TRIANGLE





0.5 × (0,0,10) + 0.4 × (10,0,0) + 0.1 × (0,-10,0)

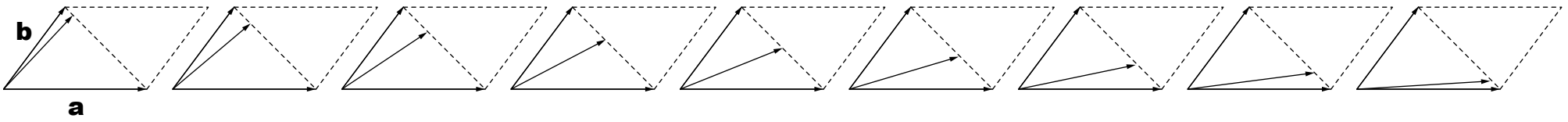
= (4,-1,5)



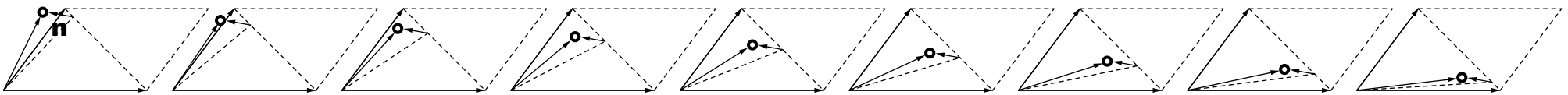
EXAMPLE OF INTERPOLATION

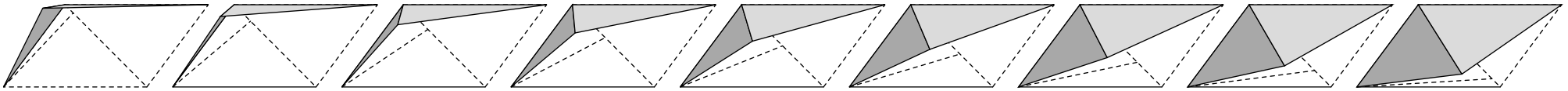
t x **a** + (1-t) x **b**

t = 0.1	t = 0.2	t = 0.3	t = 0.4	t = 0.5	t = 0.6	t = 0.7	t = 0.8	t = 0.9
0.1**a**+0.9**b**	0.2**a**+0.8**b**	0.3**a**+0.7**b**	0.4**a**+0.6**b**	0.5**a**+0.5**b**	0.6**a**+0.4**b**	0.7**a**+0.3**b**	0.8**a**+0.2**b**	0.9**a**+0.1**b**



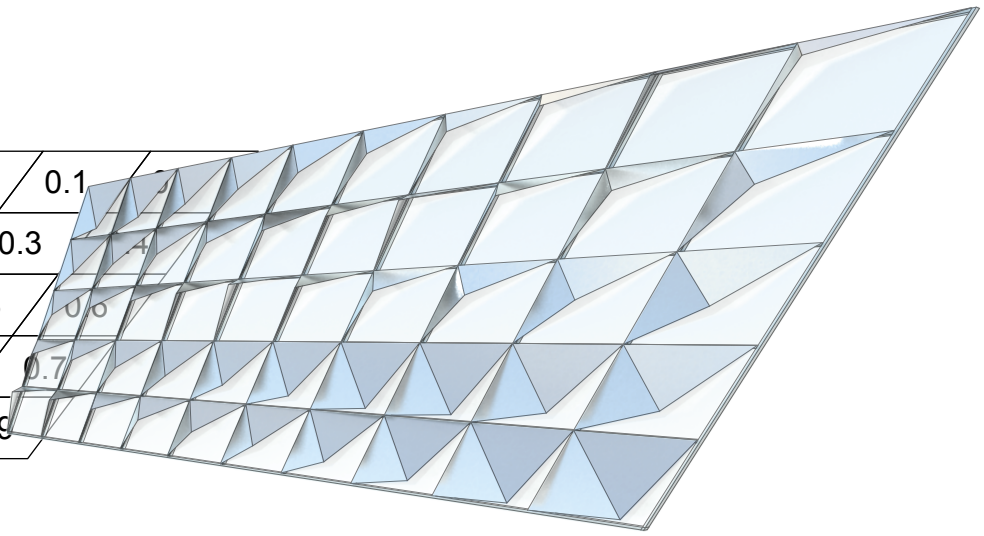
0.1**a**+0.9**b**+**n**	0.2**a**+0.8**b**+**n**	0.3**a**+0.7**b**+**n**	0.4**a**+0.6**b**+**n**	0.5**a**+0.5**b**+**n**	0.6**a**+0.4**b**+**n**	0.7**a**+0.3**b**+**n**	0.8**a**+0.2**b**+**n**	0.9**a**+0.1**b**+**n**



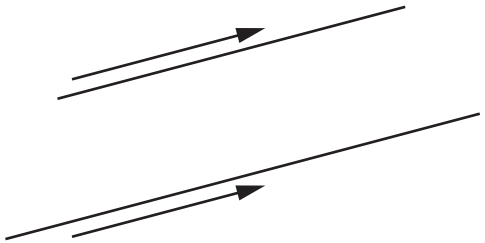


t = ?

	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.2	0.3	
0.5	0.4	0.3	0.2	0.1	0.2	0.3	0.4	0.5	
0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.9	0.8	
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	

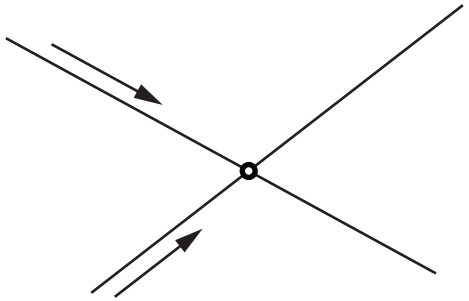


RELATIONSHIP OF INFINITE LINES



PARALELL

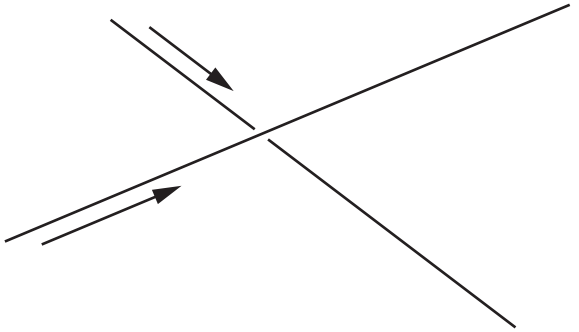
Directions (of vector) are same



INTERSECTING

Directions (of vector) are different

One shared point

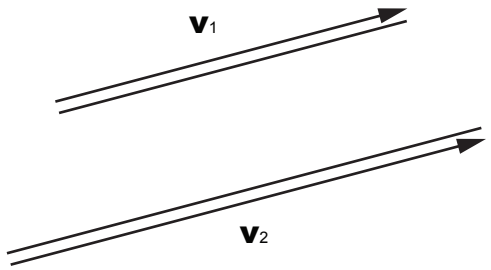


SKEW

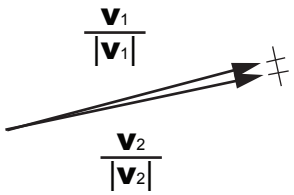
Directions (of vector) are different

No shared point

CHECKING PARALLEL



PARALELL?

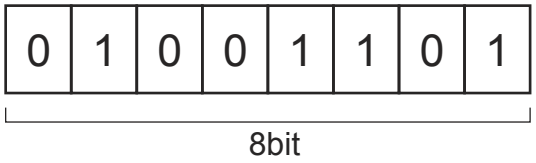


distance < tolerance ?

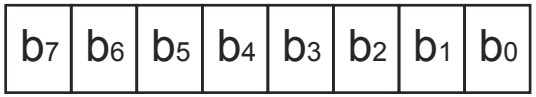
Comparing Unitized Vector	
$\frac{\mathbf{v}_1}{\lvert\mathbf{v}_1\rvert} = \frac{\mathbf{v}_2}{\lvert\mathbf{v}_2\rvert}$ or $\frac{\mathbf{v}_1}{\lvert\mathbf{v}_1\rvert} = -\frac{\mathbf{v}_2}{\lvert\mathbf{v}_2\rvert}$	$\left\lvert\frac{\mathbf{v}_1}{\lvert\mathbf{v}_1\rvert} - \frac{\mathbf{v}_2}{\lvert\mathbf{v}_2\rvert}\right\rvert <$ tolerance or $\left\lvert\frac{\mathbf{v}_1}{\lvert\mathbf{v}_1\rvert} + \frac{\mathbf{v}_2}{\lvert\mathbf{v}_2\rvert}\right\rvert <$ tolerance
Comparing Dot Product	
$\mathbf{v}_1 \cdot \mathbf{v}_2 = \lvert\mathbf{v}_1\rvert \lvert\mathbf{v}_2\rvert$ or $\mathbf{v}_1 \cdot \mathbf{v}_2 = -\lvert\mathbf{v}_1\rvert \lvert\mathbf{v}_2\rvert$	$\frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{\lvert\mathbf{v}_1\rvert \lvert\mathbf{v}_2\rvert} - 1 <$ tolerance or $\frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{\lvert\mathbf{v}_1\rvert \lvert\mathbf{v}_2\rvert} + 1 <$ tolerance
Comparing Cross Product	
$\lvert\mathbf{v}_1 \times \mathbf{v}_2\rvert = 0$	$\lvert\mathbf{v}_1 \times \mathbf{v}_2\rvert <$ tolerance
In Parfect World (With Zero Tolerance)	**In Digital World (With Tolerance)**

ERROR AND TOLELANCE : BINARY NUMBER

BINARY NUMBER



INTEGER



$$b_7 \times 2^7 + b_6 \times 2^6 + b_5 \times 2^5 + b_4 \times 2^4 + b_3 \times 2^3 + b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0$$

$$= \sum_{i=0}^{7} b_i \times 2^i$$

$$0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

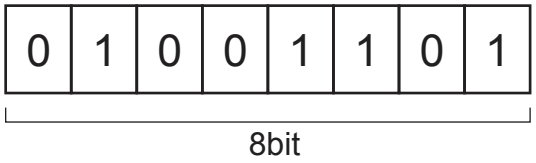
$$= 0 \times 128 + 1 \times 64 + 0 \times 32 + 0 \times 16 + 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1$$

$$= 64 + 8 + 4 + 1$$

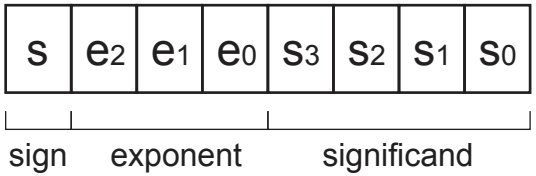
$$= 77$$

ERROR AND TOLELANCE : BINARY NUMBER

BINARY NUMBER



FLOATING POINT NUMBER



e_2	e_1	e_0	
0	0	0	0 - 3 = -3
0	0	1	1 - 3 = -2
0	1	0	2 - 3 = -1
0	1	1	3 - 3 = 0
1	0	0	4 - 3 = 1
1	0	1	5 - 3 = 2
1	1	0	6 - 3 = 3
1	1	1	7 - 3 = 4

$$\text{sign} = (-1)^s$$

$$\text{exponent} = e_2 \times 2^2 + e_1 \times 2 + e_0 - 3 = \sum_{i=0}^{2} e_i \times 2_i - 3$$

$$\text{significand} = 1 + s_3 \times 1/2 + s_2 \times 1/2^2 + s_1 \times 1/2^3 + s_0 \times 1/2^4 = 1 + \sum_{i=0}^{3} s_i \times 2^{i-4}$$

$$\text{sign} \times \text{significand} \times 2^{\text{exponent}}$$

$$(-1)^0 \times (1 + 1 \times 1/2 + 1 \times 1/2^2 + 0 \times 1/2^3 + 1 \times 1/2^4) \times 2^{(1x2^2 + 0x2 + 0x1 - 3)}$$

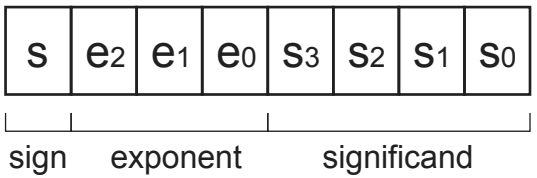
$$= 1 \times (1 + 1 \times 0.5 + 1 \times 0.25 + 0 \times 0.125 + 1 \times 0.0625) \times 2^1$$

$$= 1.8125 \times 2$$

$$= 3.625$$

ERROR & TOLERANCE:FLOATING POINT NUMBER

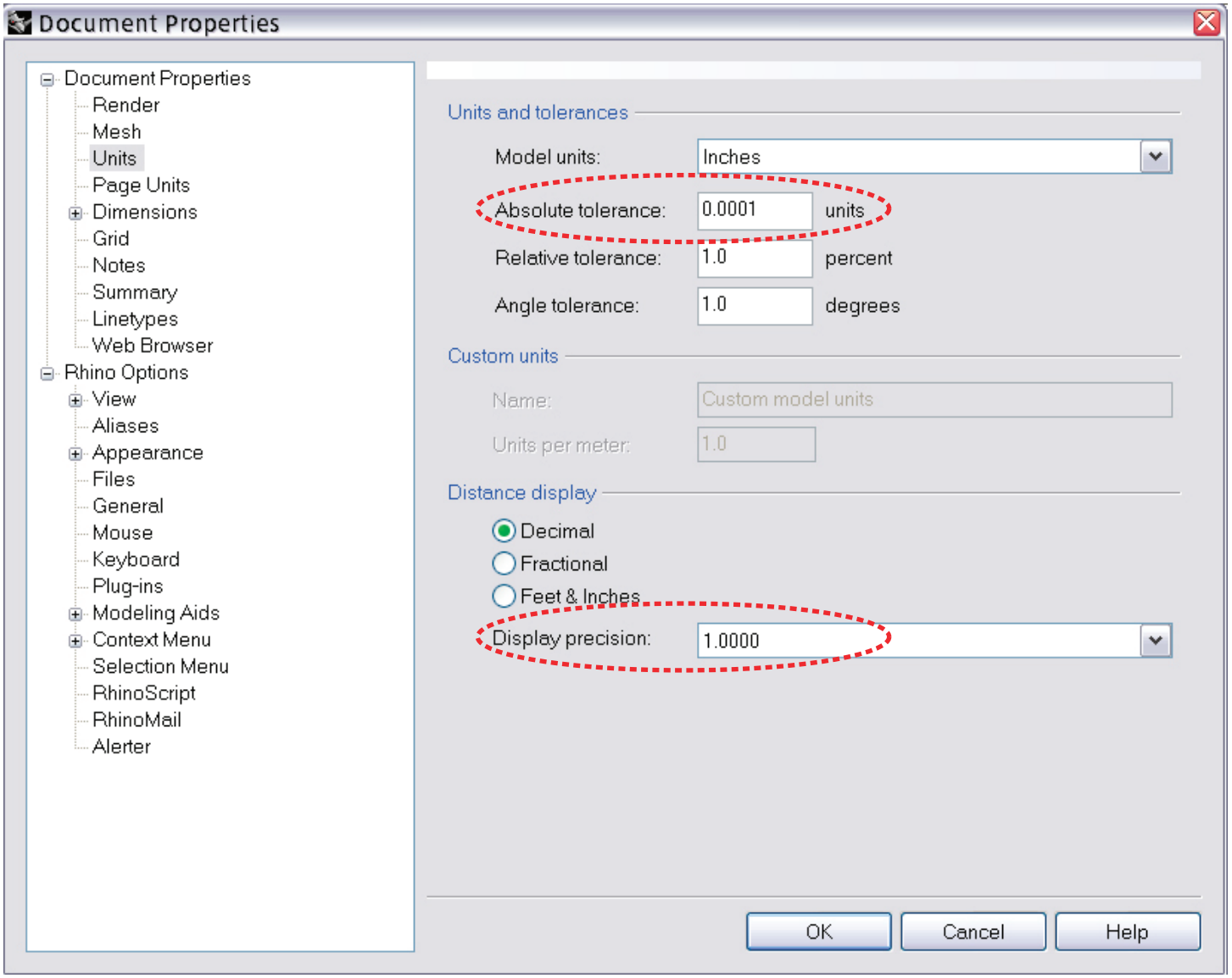
8 Bit Floating Point Number



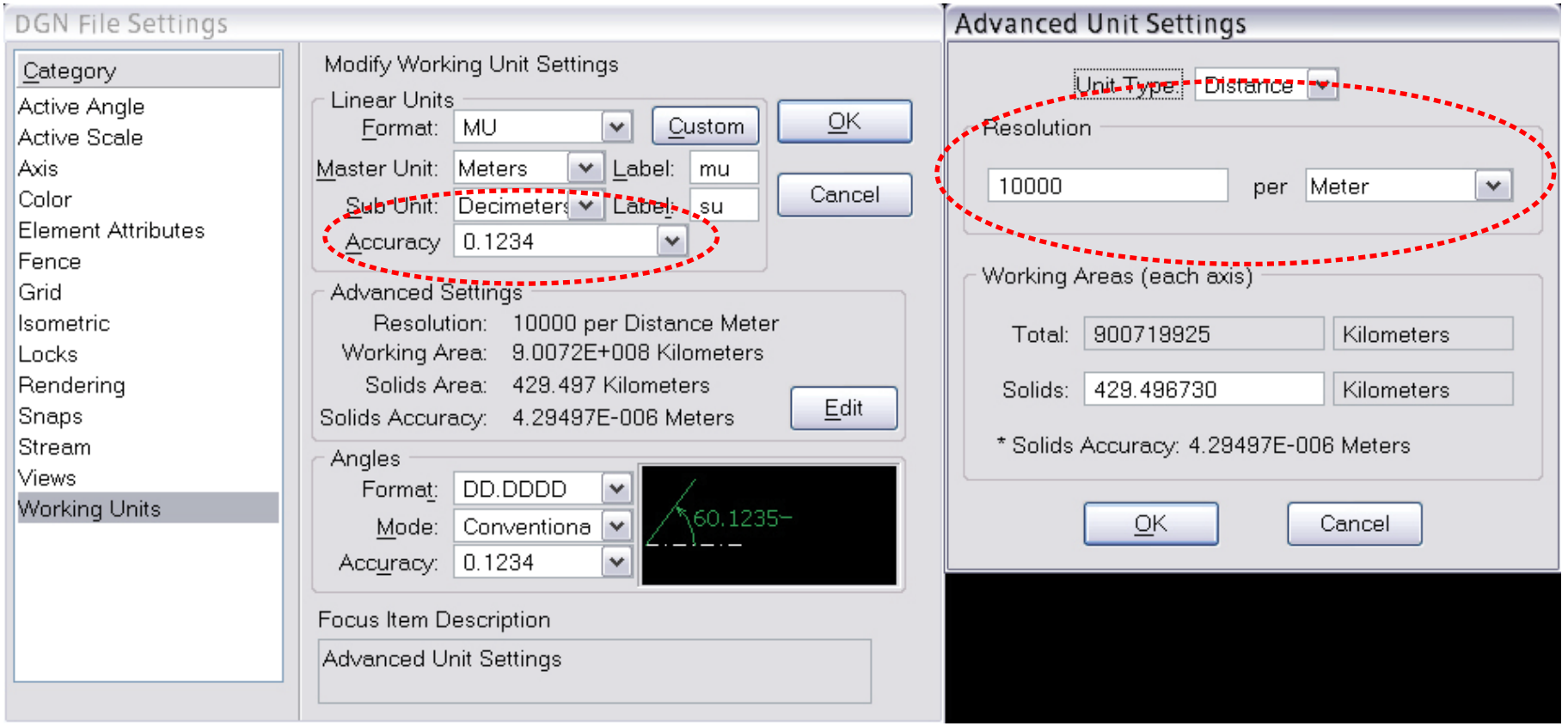
$$(-1)^{s} \times (1 + s_3 \times 1/2 + s_2 \times 1/2^2 + s_1 \times 1/2^3 + s_0 \times 1/2^4) \times 2^{(e_2 \times 2^2 + e_1 \times 2 + e_0 - 3)}$$

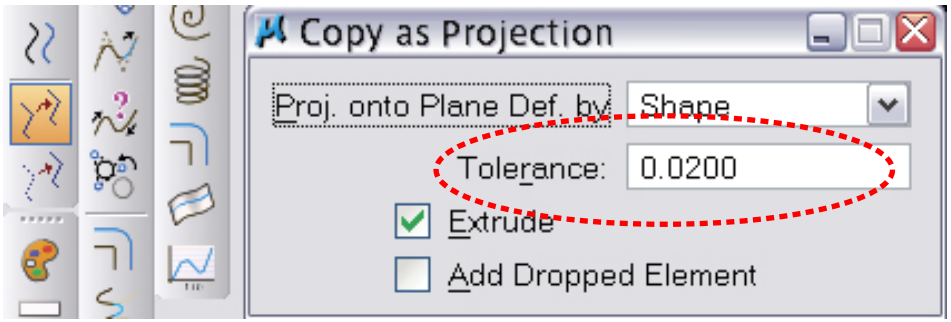
sign+exponent \ significand	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
0000	0.125	0.1328125	0.140625	0.1484375	0.15625	0.1640625	0.171875	0.1796875	0.1875	0.1953125	0.203125	0.2109375	0.21875	0.2265625	0.234375	0.2421875
0001	0.25	0.265625	0.28125	0.296875	0.3125	0.328125	0.34375	0.359375	0.375	0.390625	0.40625	0.421875	0.4375	0.453125	0.46875	0.484375
0010	0.5	0.53125	0.5625	0.59375	0.625	0.65625	0.6875	0.71875	0.75	0.78125	0.8125	0.84375	0.875	0.90625	0.9375	0.96875
0011	1	1.0625	1.125	1.1875	1.25	1.3125	1.375	1.4375	1.5	1.5625	1.625	1.6875	1.75	1.8125	1.875	1.9375
0100	2	2.125	2.25	2.375	2.5	2.625	2.75	2.875	3	3.125	3.25	3.375	3.5	3.625	3.75	3.875
0101	4	4.25	4.5	4.75	5	5.25	5.5	5.75	6	6.25	6.5	6.75	7	7.25	7.5	7.75
0110	8	8.5	9	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15	15.5
0111	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1000	-0.125	-0.1328125	-0.140625	-0.1484375	-0.15625	-0.1640625	-0.171875	-0.1796875	-0.1875	-0.1953125	-0.203125	-0.2109375	-0.21875	-0.2265625	-0.234375	-0.2421875
1001	-0.25	-0.265625	-0.28125	-0.296875	-0.3125	-0.328125	-0.34375	-0.359375	-0.375	-0.390625	-0.40625	-0.421875	-0.4375	-0.453125	-0.46875	-0.484375
1010	-0.5	-0.53125	-0.5625	-0.59375	-0.625	-0.65625	-0.6875	-0.71875	-0.75	-0.78125	-0.8125	-0.84375	-0.875	-0.90625	-0.9375	-0.96875
1011	-1	-1.0625	-1.125	-1.1875	-1.25	-1.3125	-1.375	-1.4375	-1.5	-1.5625	-1.625	-1.6875	-1.75	-1.8125	-1.875	-1.9375
1100	-2	-2.125	-2.25	-2.375	-2.5	-2.625	-2.75	-2.875	-3	-3.125	-3.25	-3.375	-3.5	-3.625	-3.75	-3.875
1101	-4	-4.25	-4.5	-4.75	-5	-5.25	-5.5	-5.75	-6	-6.25	-6.5	-6.75	-7	-7.25	-7.5	-7.75
1110	-8	-8.5	-9	-9.5	-10	-10.5	-11	-11.5	-12	-12.5	-13	-13.5	-14	-14.5	-15	-15.5
1111	-16	-17	-18	-19	-20	-21	-22	-23	-24	-25	-26	-27	-28	-29	-30	-31

ERROR AND TOLELANCE : RHINOCEROS



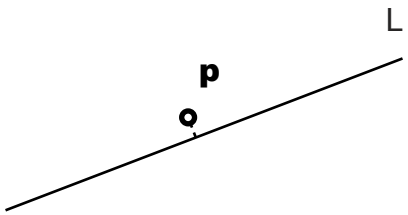
ERROR AND TOLELANCE : MICROSTATION





POINT ON LINE / DISTANCE OF POINT&LINE

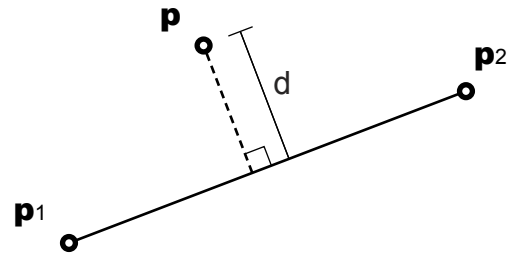
Point on Line



Is point **p** on line L?

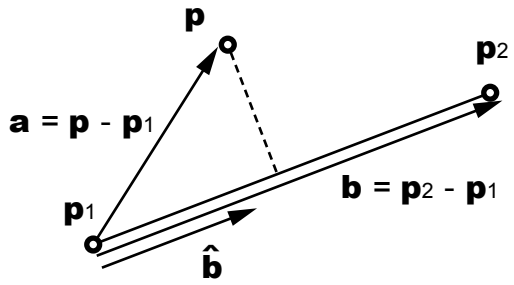
⇕

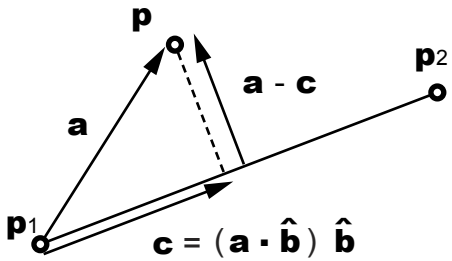
distance between **p** and L < tolerance



Distance between Point and Line (= Finding Closest Point On Line = Projection of Point to Line)

By Vector

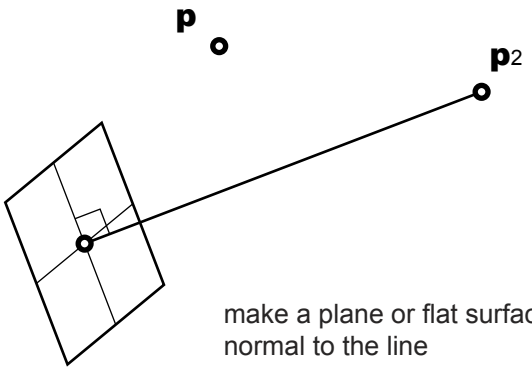




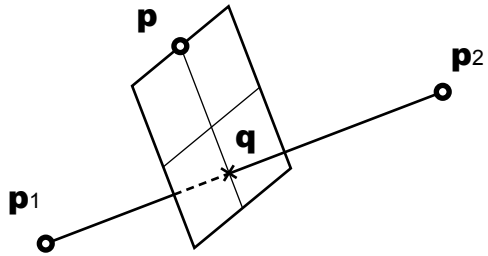
$d = |\mathbf{a} - \mathbf{c}|$

$= |\mathbf{a} - (\mathbf{a} \cdot \hat{\mathbf{b}})\,\hat{\mathbf{b}}|$

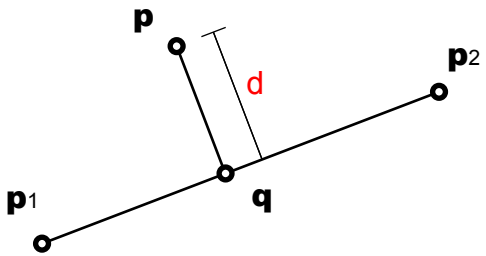
By Geometry



make a plane or flat surface normal to the line



move the plane or the surface to the point and take intersection of it and the line



measure distance between **p** and **q**

By Formula

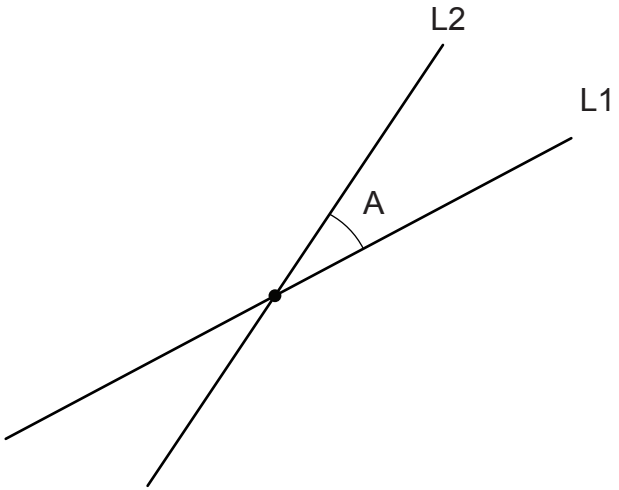
Line : $\dfrac{x - q_x}{a} = \dfrac{y - q_y}{b} = \dfrac{z - q_z}{c}$

Point : $p = (p_x, p_y, p_z)$

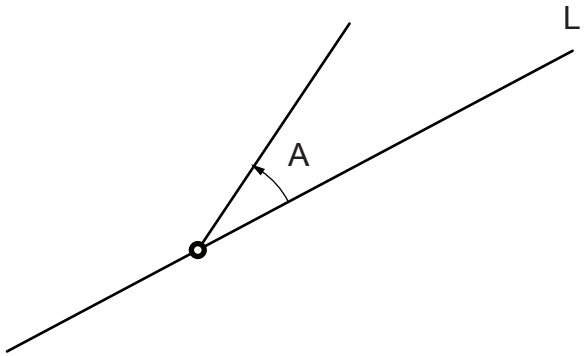
when $x' = p_x - q_x \quad y' = p_y - q_y \quad z' = p_z - q_z$

$$d = \sqrt{\frac{(b^2 + c^2)\,x'^2 + (c^2 + a^2)\,y'^2 + (a^2 + b^2)\,z'^2 - 2abx'y' - 2bcy'z' - 2caz'x'}{a^2 + b^2 + c^2}}$$

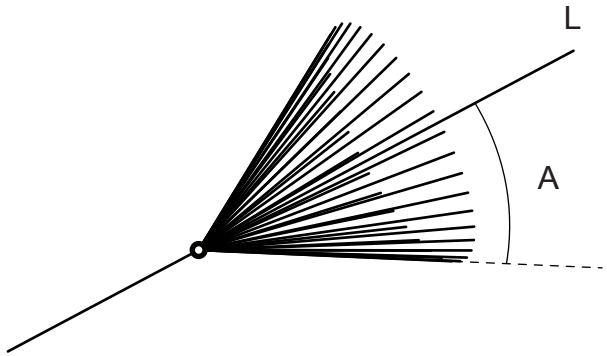
ANGLE TOWARD LINE



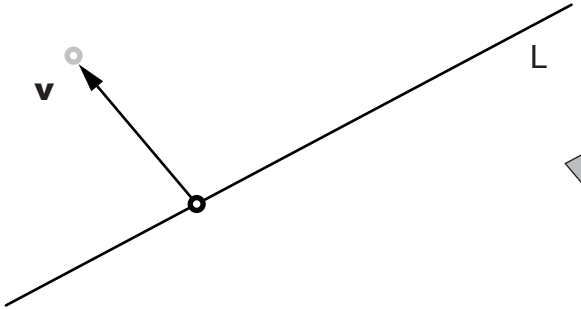
Measuring Angle of Line L1 and L2



Line having angle A to L

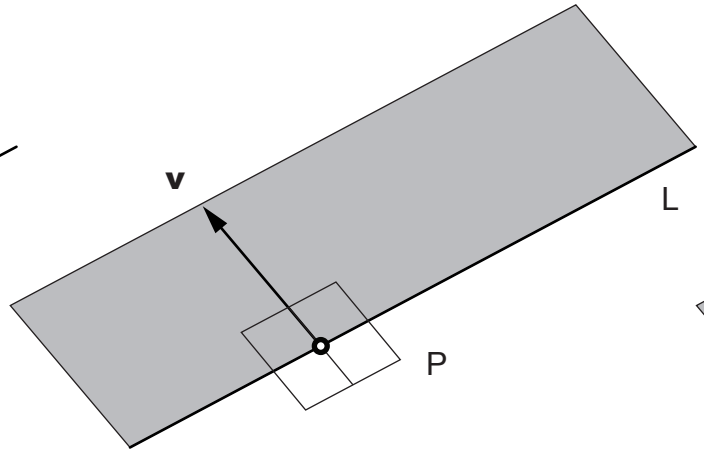


Possible Line having angle A to L

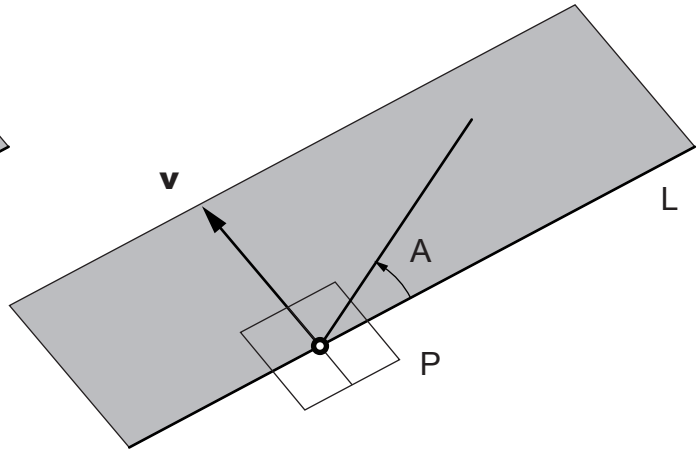


Define Direction by Vector **v**

(Or by Point and Creating Vector out of it)



Plane P Defined by Line L and Vector **v**

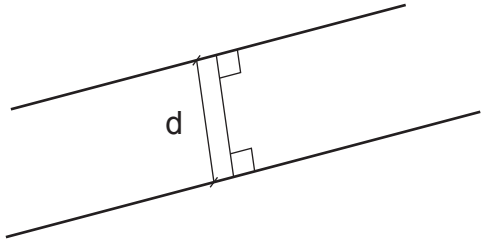


Rotate on Plane P

DISTANCE OF 2 LINES

Distance of 2 lines = Minimum distance between all possible pair of points on each line

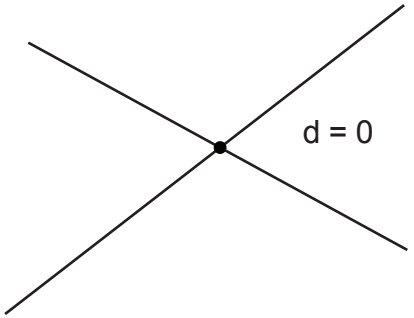
PARALELL LINES



Distance is equals to length of line perpendicular to both lines.

Such line can exist anywhere.

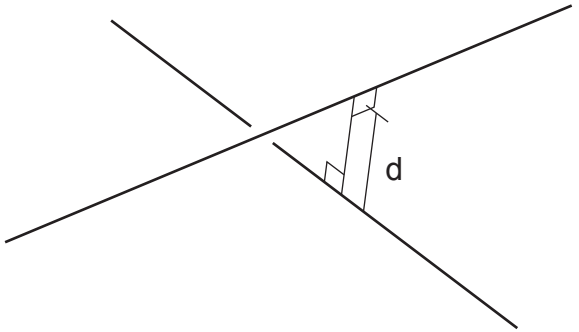
INTERSECTING LINES



Distance is zero at intersection.

Only one intersection can exist.

SKEW LINES

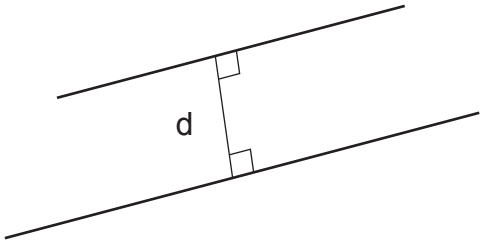


Distance is equals to length of line perpendicular both lines.

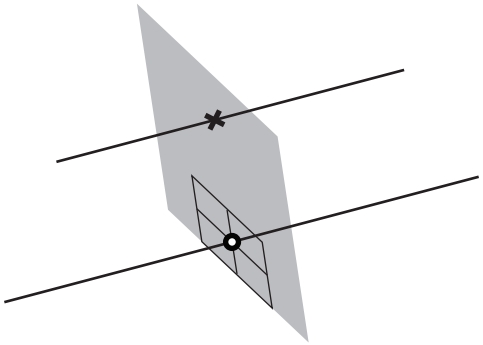
Only one such line can exist.

DISTANCE OF PARALLEL LINES

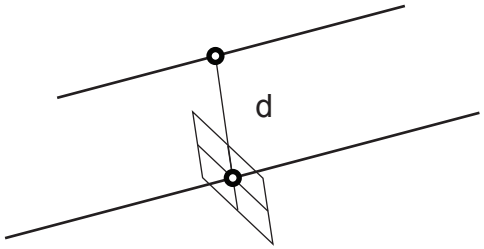
PARALELL LINES



By Plane and Intersection

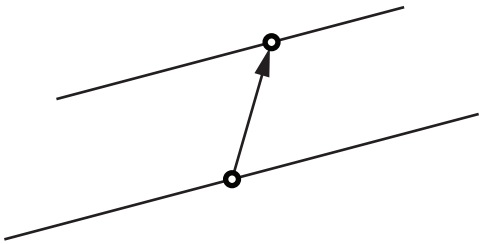


make a plane parpendicular to a line and take intersection

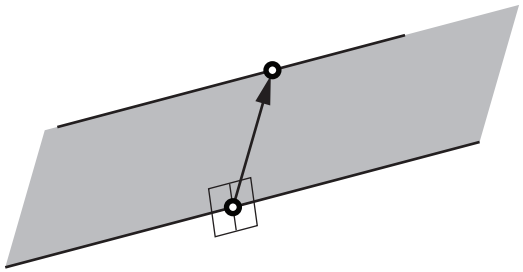


measure distance between the original pointand the intersection point

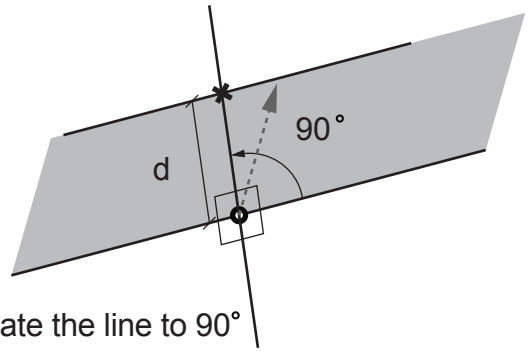
By Vector, Plane and Rotation



take any point on each line and create a vector

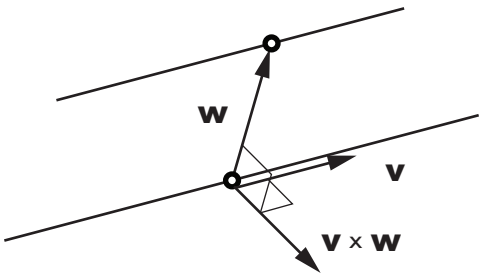


make plane with the line and the vector

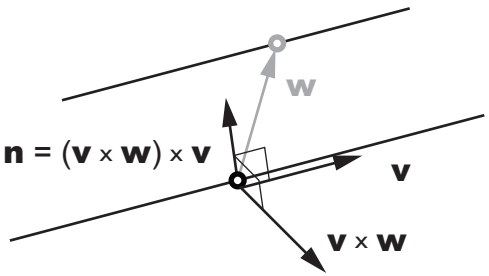


rotate the line to 90° and take intersection

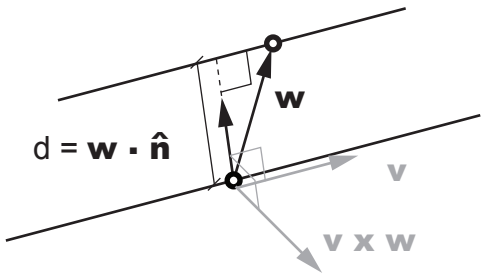
By Vector, Cross Product, Dot Product



take cross product of the line vector **v** and the vector between two line **w**



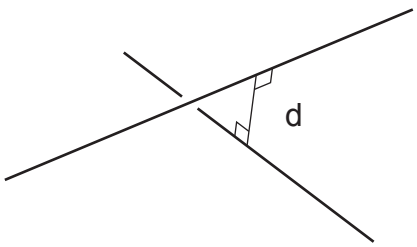
take cross product of the cross product and **v**



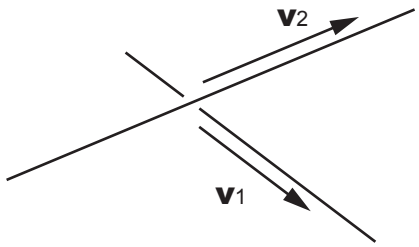
take dot product of w and unitized second cross product

DISTANCE OF SKEW LINES

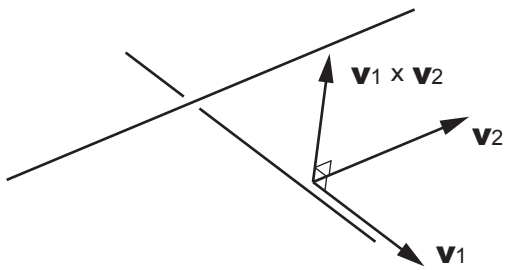
SKEW LINES



1. Cross Product

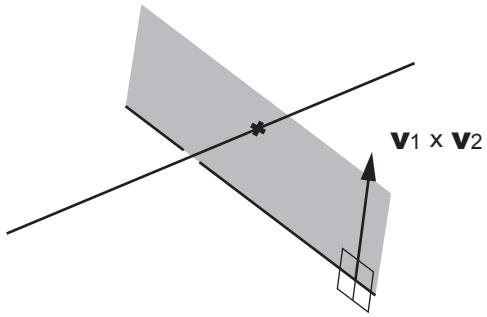


take two vectors of each line

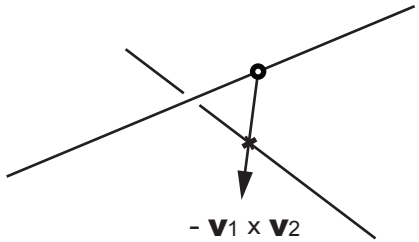


take cross product

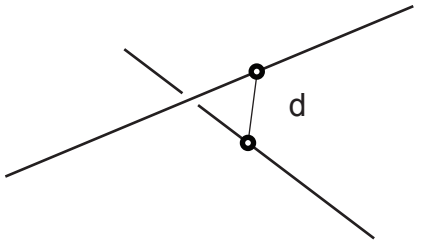
2A. Pland and Intersection



create a plane by the line and the cross product and take intersection

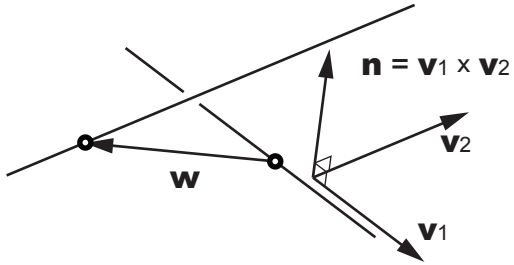


create a line toward the opposite direction of the cross vector and take intersection

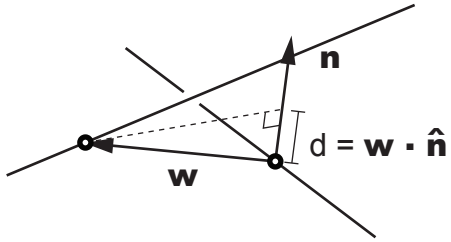


measure two intersection point

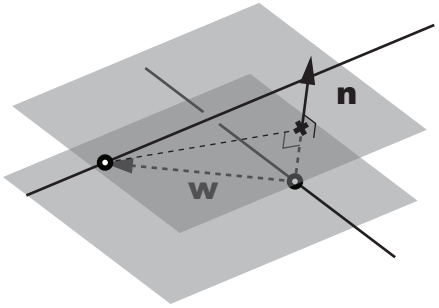
2B. Dot Product



take a point on each line and create a vector **w**

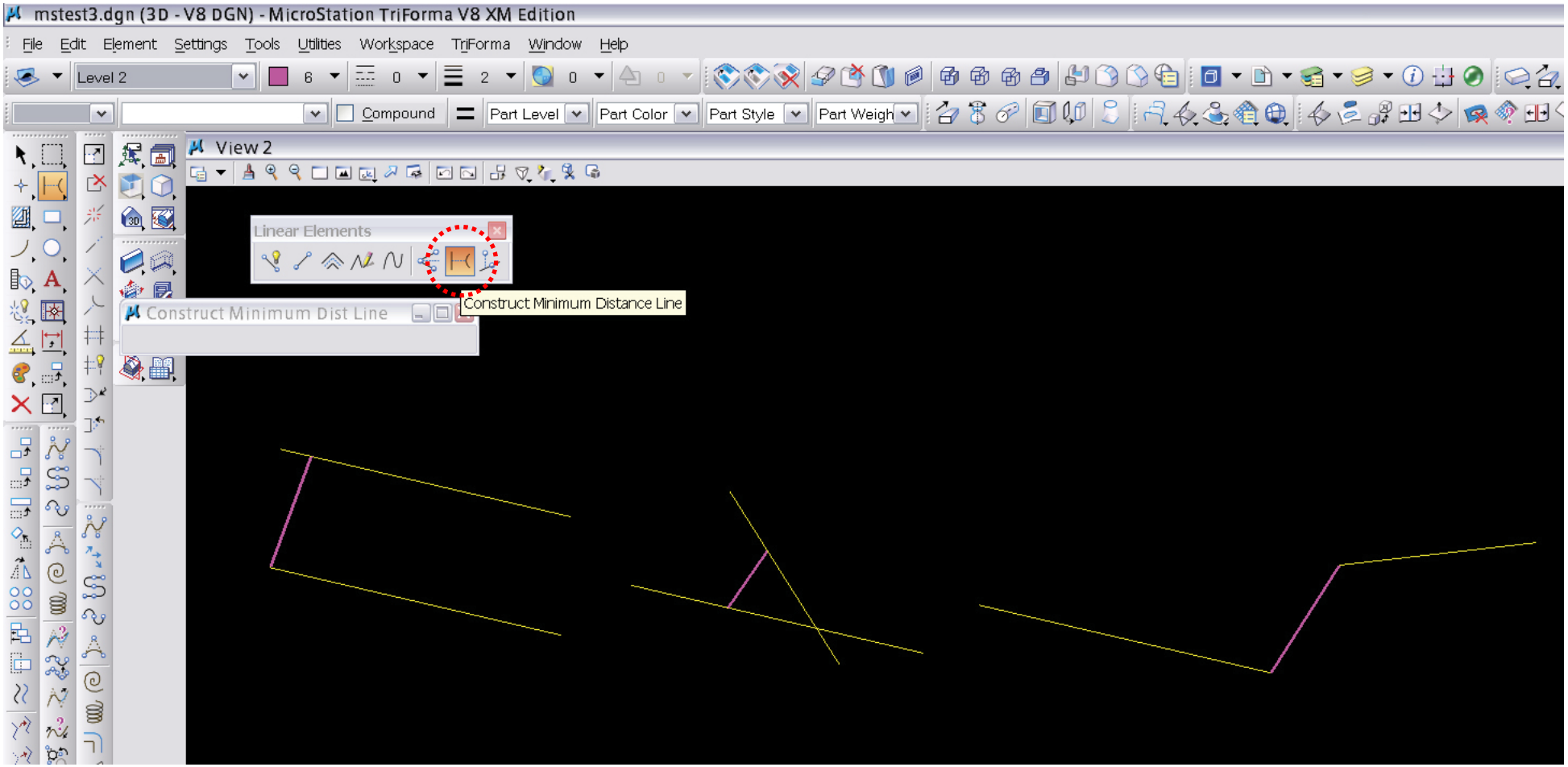


take dot product of **w** and unitized **n** and it's equals to d

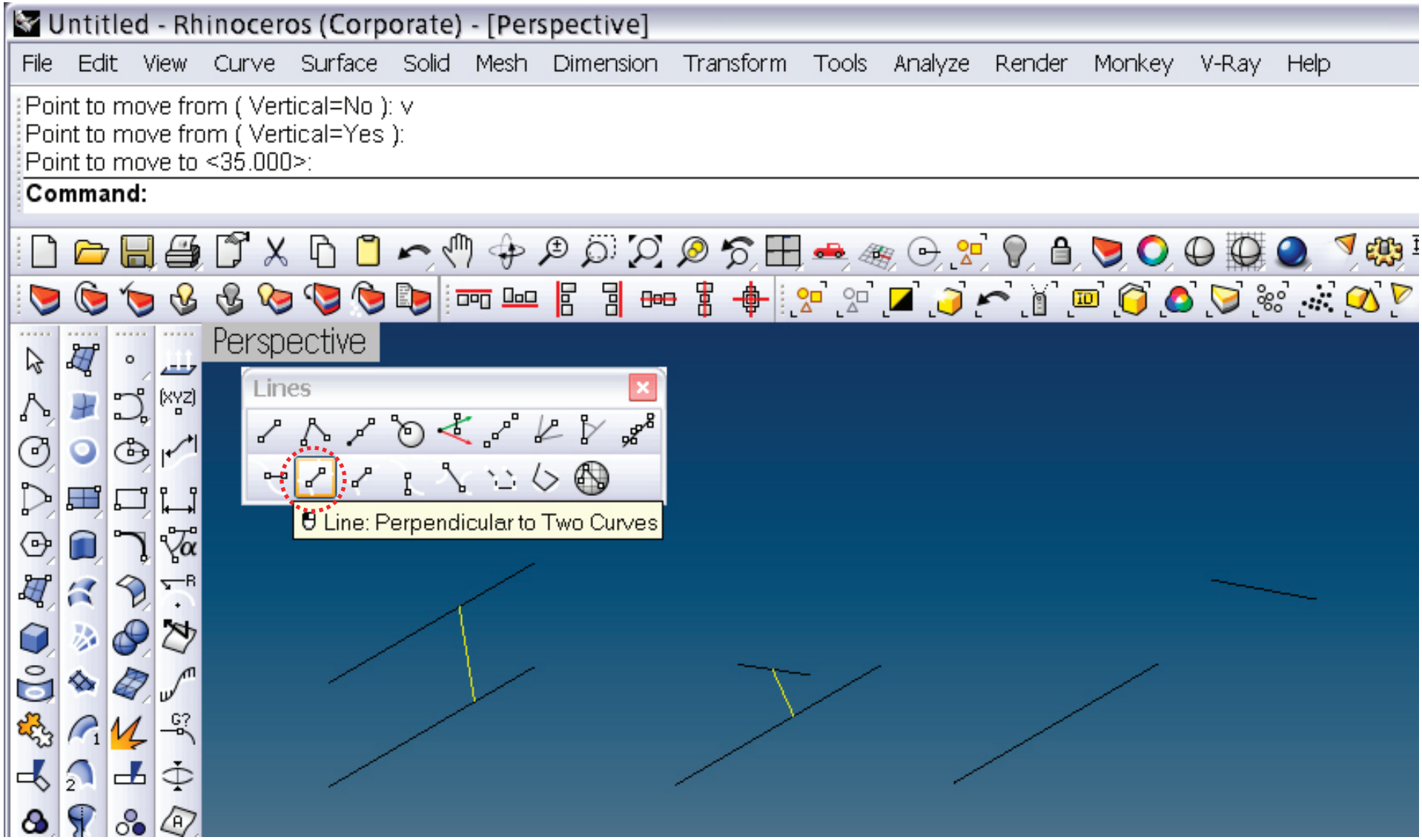


n is normal vector of parallel planes on which each line lies

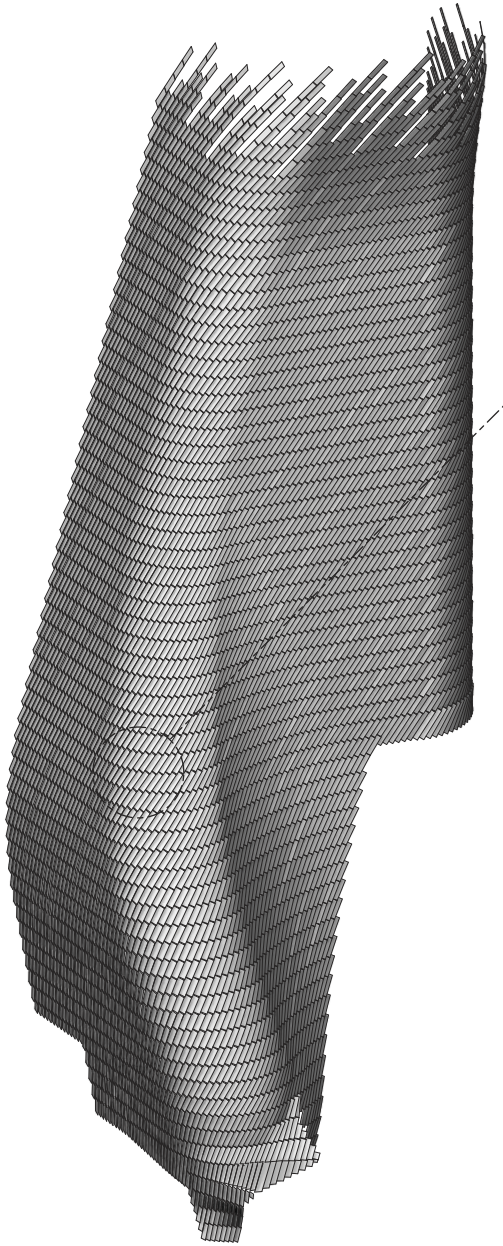
DISTANCE OF 2 LINES : MICROSTATION

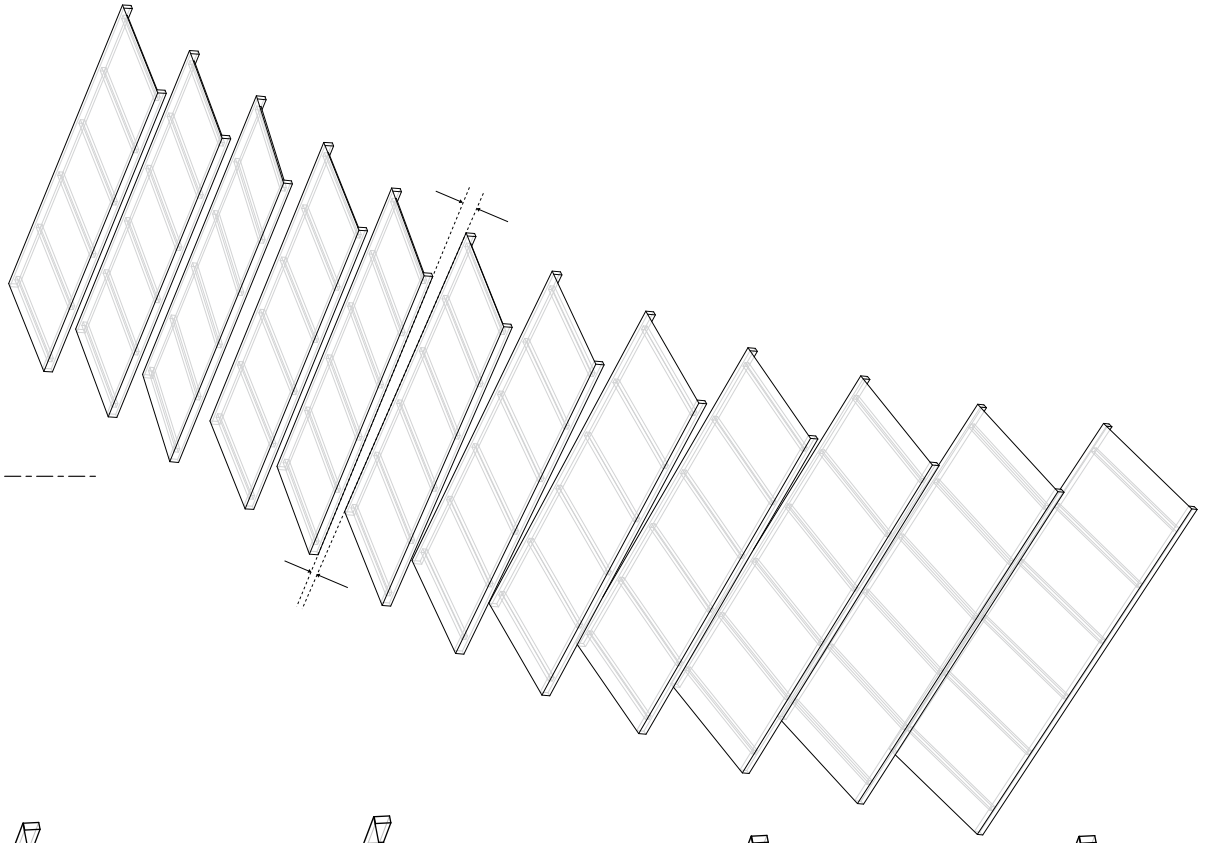


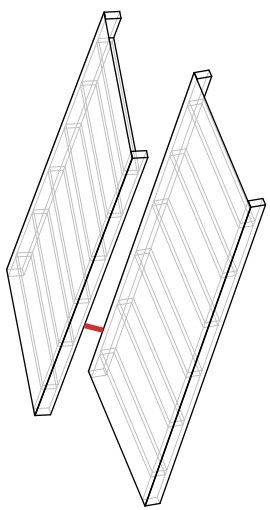
DISTANCE OF 2 LINES : RHINOCEROS



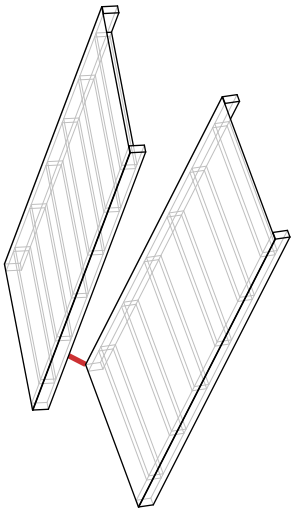
EXAMPLE: DISTANCE OF SKEW LINES



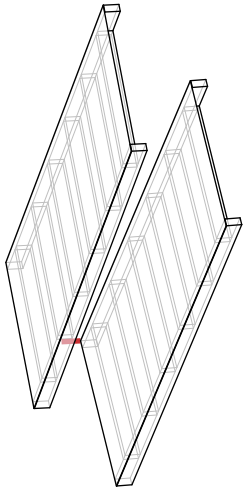




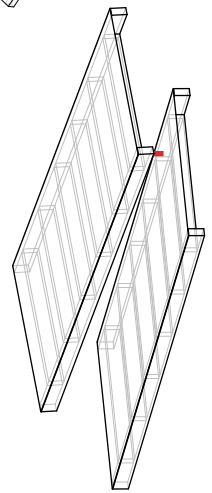
skew edges



edge to corner

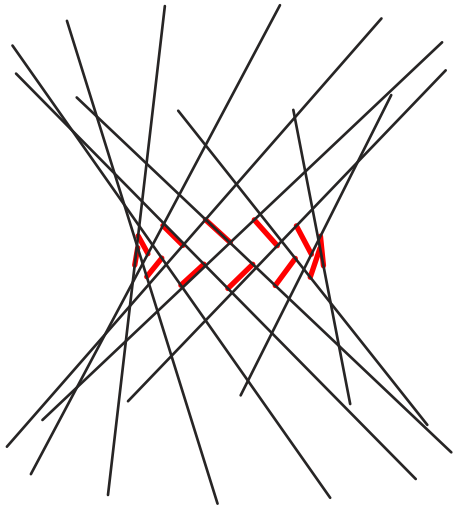


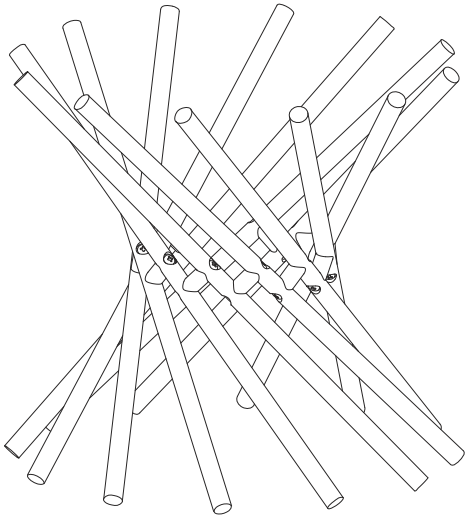
corner to back face

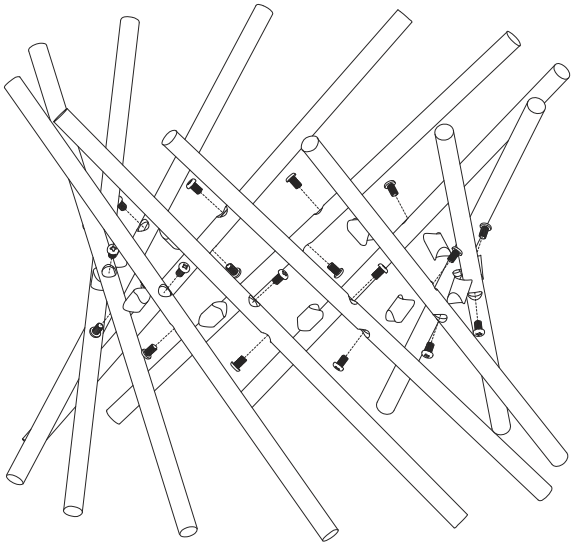


corner to front face

EXAMPLE : RATIONALITY OF CONNECTED SKEW LINES



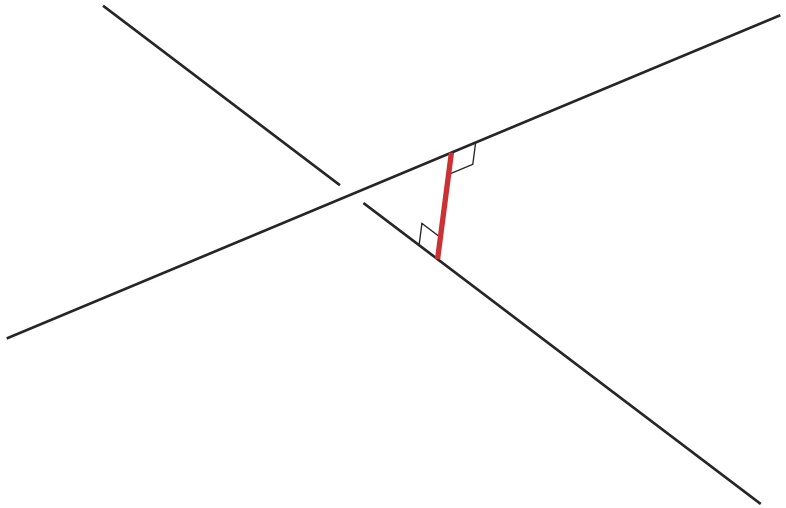




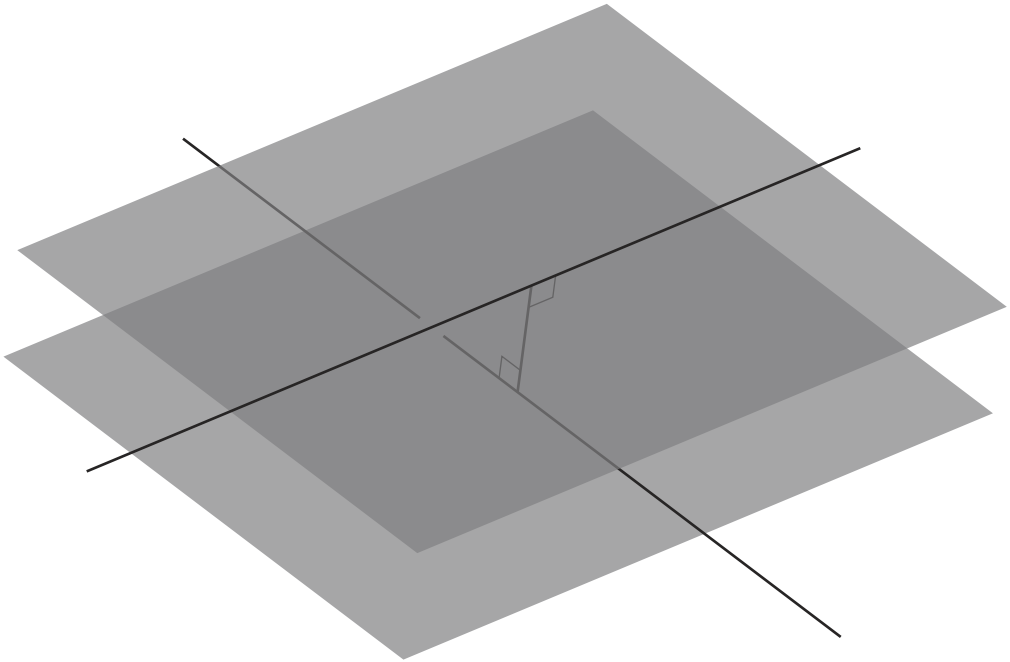
EXAMPLE : CONNECTING SKEW LINES

Installation by Ai Weiwei and Herzog & de Meuron

EXERCISE : SHORTEST LINE BETWEEN SKEW LIENS



Draw skew lines and the shortest line between them without using “Minimum Distance Line” tool nor “Line Perpendicular to Two Curve” tool.



And make two parallel planes on which each skew line lie.