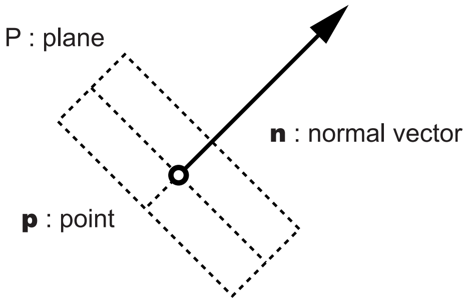
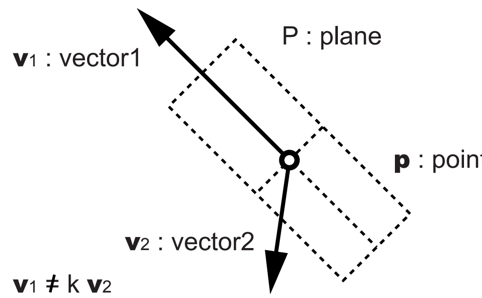
PLANE

PLANE BY A POINT AND A NORMAL VECTOR

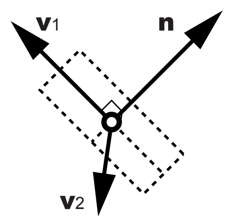


PLANE BY A POINT AND TWO VECTORS



$\mathbf{v}_1 \neq k\,\mathbf{v}_2$

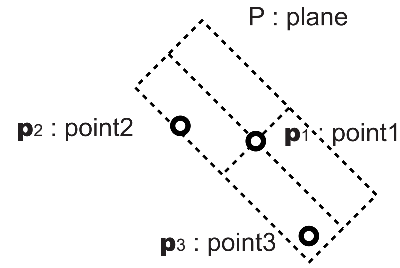
($\mathbf{v}_1$ and $\mathbf{v}_2$ cannot be parallel)



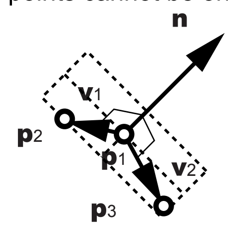
$\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2$

(cross vector)

PLANE BY 3 POINTS



(3 points cannot be on a line)



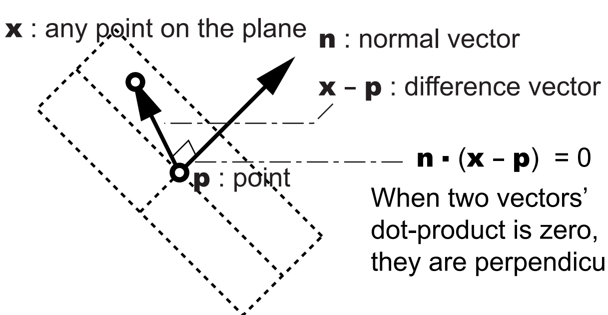
$\mathbf{v}_1 = \mathbf{p}_2 - \mathbf{p}_1$

$\mathbf{v}_2 = \mathbf{p}_3 - \mathbf{p}_1$

$\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2$

$= (\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)$

EQUATIONS



Polynomial Equation	$a x + b y + c z + d = 0$
Vector Equation	$\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

Relationship of Polynomial and Vector Equation

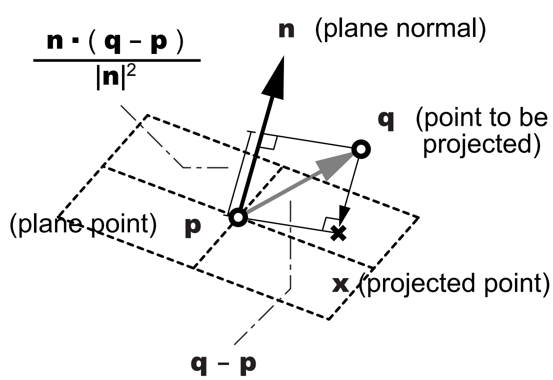
if $\mathbf{n} = (n_x, n_y, n_z)$ $\quad \mathbf{p} = (p_x, p_y, p_z)$

$$\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) =$$

$$n_x x + n_x y + n_z z - n_x p_x - n_y p_y - n_z p_z = 0$$

$$a x + b y + c z + d = 0$$

PROJECTION ONTO PLANE IN NORMAL DIRECTION



PLANE: $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$\mathbf{x} = ?$

$\mathbf{x} = \mathbf{q} + k\,\mathbf{n}$

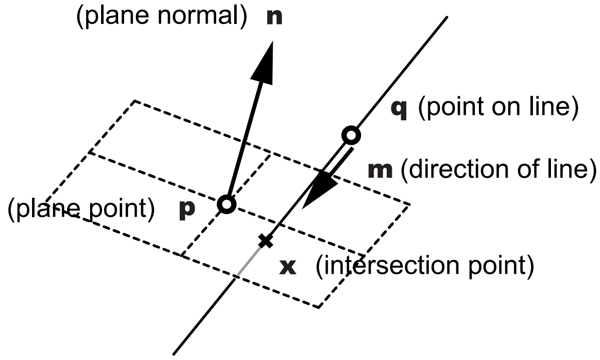
$\mathbf{n} \cdot ((\mathbf{q} + k\,\mathbf{n}) - \mathbf{p}) = 0$

$$k = \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{q})}{|\mathbf{n}|^2}$$

$$\mathbf{x} = \mathbf{q} + \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{q})}{|\mathbf{n}|^2}\,\mathbf{n}$$

PLANE

INTERSECTION OF PLANE AND LINE / PROJECTION OF POINT ONTO PLANE IN SPECIFIED DIRECTION



PLANE: $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$\mathbf{x} = ?$

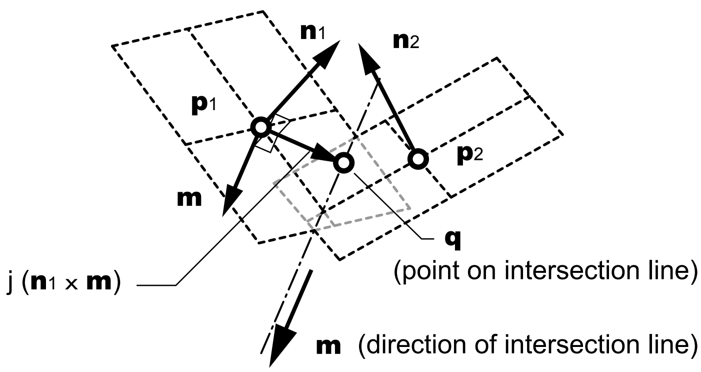
LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

$\mathbf{n} \cdot ((\mathbf{q} + k\,\mathbf{m}) - \mathbf{p}) = 0$

$$k = \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{q})}{\mathbf{n} \cdot \mathbf{m}}$$

$$\mathbf{x} = \mathbf{q} + \frac{\mathbf{n} \cdot (\mathbf{p} - \mathbf{q})}{\mathbf{n} \cdot \mathbf{m}}\,\mathbf{n}$$

INTERSECTION OF TWO PLANES



PLANE 1: $\mathbf{n}_1 \cdot (\mathbf{x} - \mathbf{p}_1) = 0$

PLANE 2: $\mathbf{n}_2 \cdot (\mathbf{x} - \mathbf{p}_2) = 0$

INTERSECTION LINE: $\mathbf{x} = \mathbf{q} + k\,\mathbf{m}$

$\mathbf{m} = \mathbf{n}_1 \times \mathbf{n}_2$

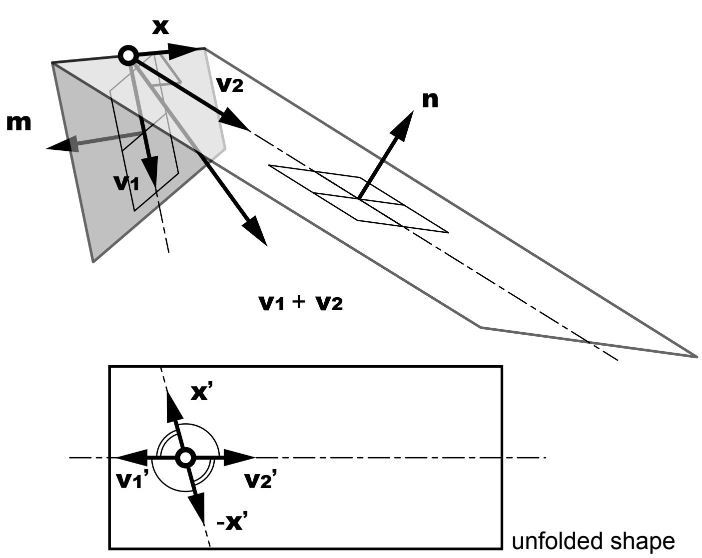
$\mathbf{q} = \mathbf{p}_1 + j\,(\mathbf{n}_1 \times \mathbf{m})$

$\mathbf{n}_2 \cdot (\mathbf{q} - \mathbf{p}_2) = 0$

$$j = \frac{\mathbf{n}_2 \cdot (\mathbf{p}_2 - \mathbf{p}_1)}{\mathbf{n}_2 \cdot (\mathbf{n}_1 \times \mathbf{m})}$$

$$\mathbf{q} = \mathbf{p}_1 + \frac{\mathbf{n}_2 \cdot (\mathbf{p}_2 - \mathbf{p}_1)}{\mathbf{n}_2 \cdot (\mathbf{n}_1 \times \mathbf{m})}\,(\mathbf{n}_1 \times \mathbf{m})$$

FOLDING VECTOR



When axis $\mathbf{v}_1$, $\mathbf{v}_2$ and normal of one plane $\mathbf{m}$ are given, what are folding vector $\mathbf{x}$ and normal of another plane $\mathbf{n}$ which makes $\mathbf{v}_1$ and $\mathbf{v}_2$ straight when they are unfolded?

$|\mathbf{v}_1| = 1, |\mathbf{v}_2| = 1, |\mathbf{x}| = 1$

$\mathbf{x} \cdot \mathbf{v}_2 = -\mathbf{x} \cdot \mathbf{v}_1$ (angle of $\mathbf{x}$&$\mathbf{v}_2$ and angle of $-\mathbf{x}$&$\mathbf{v}_1$ are same)

$\mathbf{x} \cdot (\mathbf{v}_2 + \mathbf{v}_1) = 0$

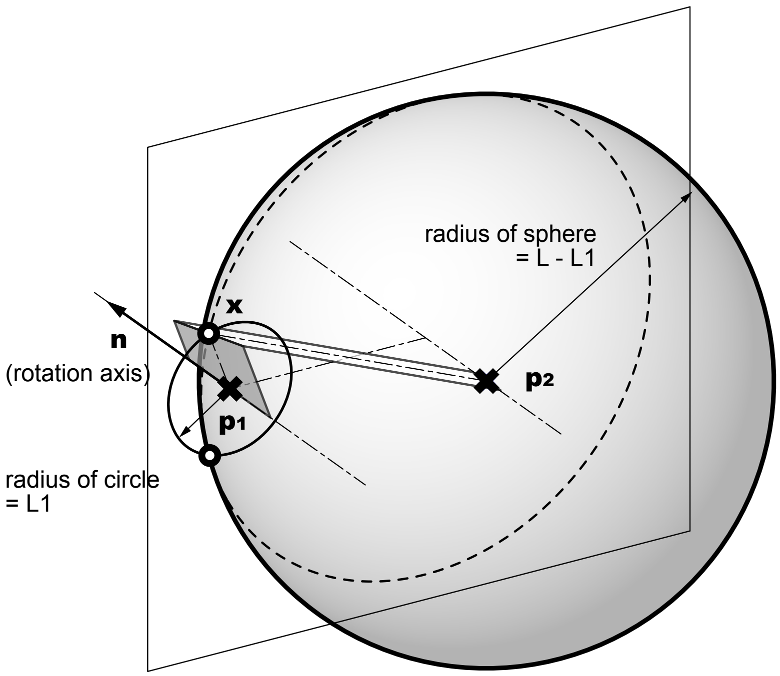
$\mathbf{x} \cdot \mathbf{m} = 0$ ($\mathbf{x}$ is perpendicular to $\mathbf{m}$ because $\mathbf{x}$ is on the plane)

$\mathbf{x} = \mathbf{m} \times (\mathbf{v}_2 + \mathbf{v}_1)$

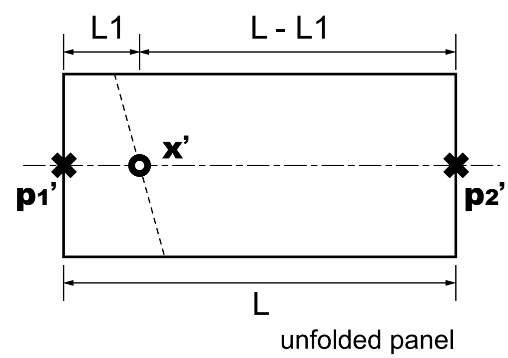
$\mathbf{n} = \mathbf{v}_2 \times (\mathbf{m} \times (\mathbf{v}_2 + \mathbf{v}_1))$

PLANE

FOLDING POINT TO KEEP UNFOLED LENGTH CONSTANT

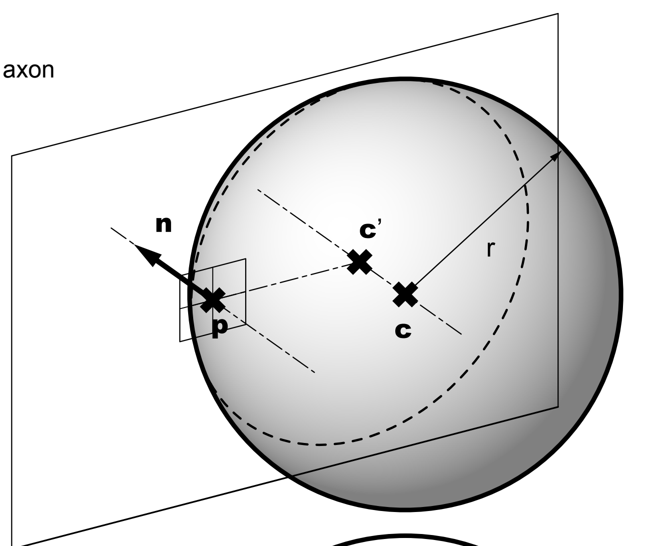


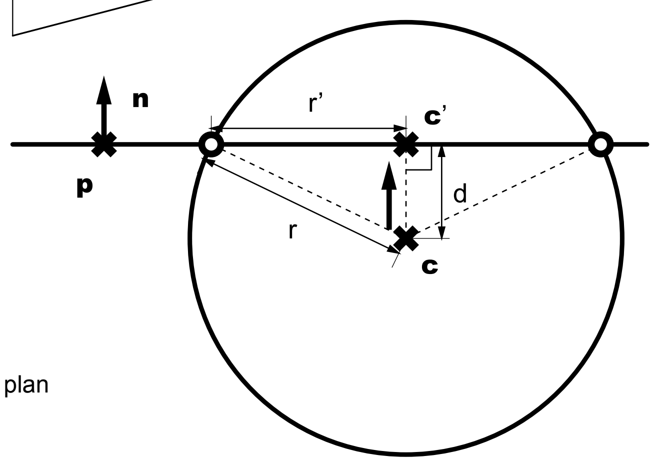
When end points of folded panel **p1** and **p2** are given, and panel length L and distance from the end point **p1** to folding point **x** is specified,folding point **x** in 3D is provided by intersection of a circle whose center is **p1**, radius is L1 and normal is rotation axis n and a sphere whose center is **p2** and radius is L-L1.



INTERSECTION OF PLANE AND SPHERE

axon





PLANE $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$$|\mathbf{n}| = 1$$

$$\mathbf{c}' = k\mathbf{n} + \mathbf{c}$$

$$\mathbf{n} \cdot (k\mathbf{n} + \mathbf{c} - \mathbf{p}) = 0$$

$$k = \mathbf{n} \cdot (\mathbf{p} - \mathbf{c})$$

$$\mathbf{c}' = (\mathbf{n} \cdot (\mathbf{p} - \mathbf{c}))\mathbf{n} + \mathbf{c}$$

$$d = |\mathbf{c}' - \mathbf{c}|$$

$$= |(\mathbf{n} \cdot (\mathbf{p} - \mathbf{c}))\mathbf{n} + \mathbf{c} - \mathbf{c}|$$

$$= (\mathbf{n} \cdot (\mathbf{p} - \mathbf{c}))$$

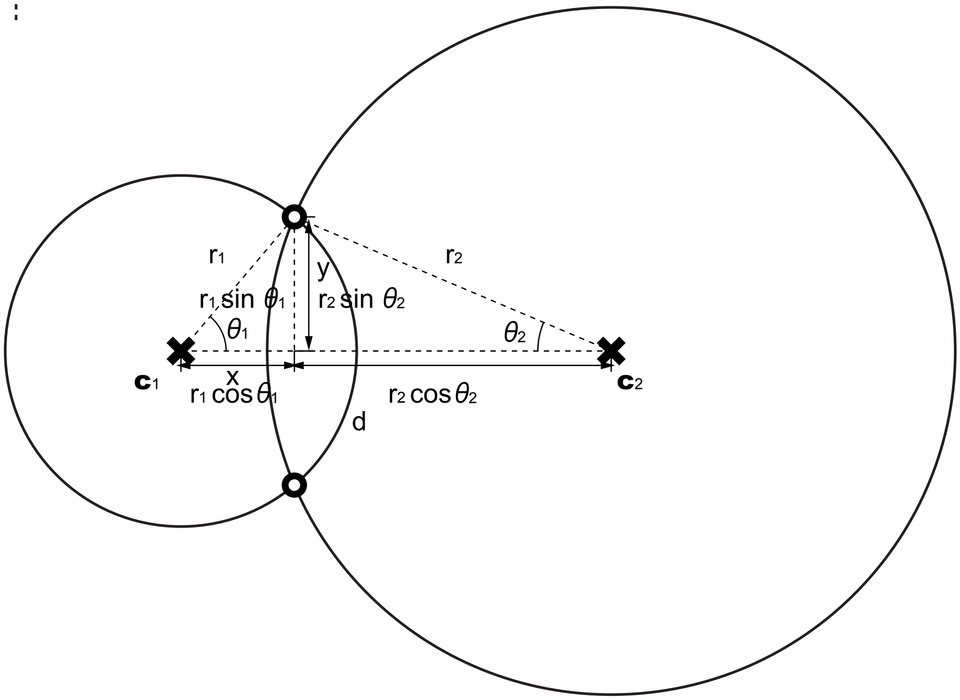
$r'^2 = r^2 - d^2$ (The Pythagorean theorem)

$$r' = \sqrt{r^2 - d^2}$$

$$r' = \sqrt{r^2 - (\mathbf{n} \cdot (\mathbf{p} - \mathbf{c}))^2}$$

PLANE

INTERSECTION OF TWO CIRCLES



$d = | \mathbf{c}_1 - \mathbf{c}_2 |$

$x = ? \qquad y = ?$

$$\begin{cases} r_1 \sin\theta_1 = r_2 \sin\theta_2 \\ r_1 \cos\theta_1 + r_2 \cos\theta_2 = d \end{cases}$$

$$\rightarrow r_1^2 \sin^2\theta_1 + r_1^2 \cos^2\theta_1 = r_2^2 \sin^2\theta_2 + r_2^2 \cos^2\theta_2 + d^2 - 2d\, r_2 \cos\theta_2$$

$$r_1^2 = r_2^2 + d^2 - 2d\, r_2 \cos\theta_2$$

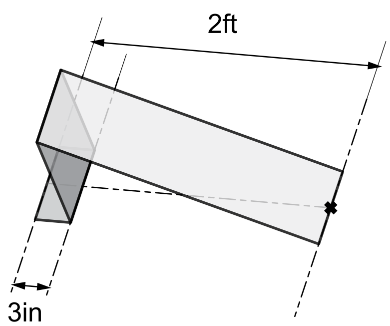
$$r_2 \cos\theta_2 = \frac{d^2 - r_1^2 + r_2^2}{2d}$$

$$x = d - r_2 \cos\theta_2$$

$$x = \frac{d^2 + r_1^2 - r_2^2}{2d}$$

$$y = \sqrt{r_1^2 - x^2}$$

EXERCISE



FOLD 3' x 1' SHEET TO HAVE THE MID POINT OF RIGHT EDGE ON THE SPECIFIED LOCATION LIKE THE LEFT DIAGRAM.